

## GALOIS GROUPS AND FUNDAMENTAL GROUPS

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**Talk 1: Infinite Galois Theory (0).** (Chapter 1.3) Introduce the notion of a profinite group (see Construction 1.3.2 and Definition 1.3.3). Prove that Galois groups are naturally profinite groups (Proposition 1.3.5). The main goal of this talk is to prove Theorem 1.3.11, the infinite analogue of the main theorem of Galois theory. If time permits also prove Lemma 1.3.8 and Corollary 1.3.9 which are necessary ingredients of the proof, otherwise just state them.

**Talk 2: Interlude on Category Theory (-).** (Chapter 1.4) Give a quick introduction to category theory following Chapter 1.4 of Szamuely's book. Define the notions of a category, functors and natural transformations and give examples. Define fully faithful and essentially surjective functors and prove Lemma 1.4.9. Discuss Example 1.4.10. After that state and prove the Yoneda Lemma (Lemma 1.4.12) and Corollary 1.4.13.

**Talk 3: Finite Étale Algebras (0).** (Chapter 1.5) The main goal of this talk is to state and understand Grothendieck's reformulation of Theorem 1.3.11 (Theorem 1.5.4). For this prove Lemma 1.5.1 and Theorem 1.5.2. Then define the notion of a finite étale algebra and conclude Theorem 1.5.4. If there is some time left discuss Proposition 1.5.6.

**Talk 4: Basics of Covering Theory (-).** (Chapter 2.1) Define the notion of a covering space (Definition 2.1.1) and give first examples (see Example 2.1.2 and Example 2.1.8). Prove Proposition 2.1.3 and Corollary 2.1.4. After that discuss Definition 2.1.6 and Lemma 2.1.7. After that discuss the remaining examples in 2.1.8. Try to draw some pictures of these coverings.

**Talk 5: Galois Covers (0).** (Chapter 2.2) Discuss Lemma 2.2.1 and Propositions 2.2.2 - 2.2.4 (you may skip some of the proofs here if you don't have the time). After that introduce the notion of a Galois cover (Definition 2.2.5) and discuss Proposition 2.2.7. Then prove Theorem 2.2.10, which is the main goal of the talk. For this you need Lemma 2.2.11 but you may skip the proof of this. Try to stress the analogy between Theorem 2.2.10 and Theorem 1.3.11.

**Talk 6: Monodromy (0).** (Chapter 2.3) Quickly recall the definition of the fundamental group of a topological space and define the notion of a simply connected topological space. Prove Lemma 2.3.2, discuss construction 2.3.3 and try to draw some pictures of the monodromy action. State Theorem 2.3.5 and Theorem 2.3.7 (the proofs will be given in the next talk) and then prove Theorem 2.3.4 using the two previous theorems. After that discuss Corollary 2.3.9. Try to highlight the similarities with Theorem 1.5.4.

**Talk 7: The Universal Cover (+).** (Chapter 2.4) Construct the universal cover (Construction 2.4.1) and then prove Theorem 2.3.5. After that the goal is to prove Theorem 2.3.7. For this you need a bunch of preliminary results (2.4.3 - 2.4.6 in the book). You may leave out some of the proofs here if you do not have the time. Then prove Theorem 2.3.7. Conclude by discussing Proposition 2.4.9 and Example 2.4.11.

**Talk 8: Locally Constant Sheaves (0).** (Chapter 2.5) Define the notions of presheaves, sheaves and stalks (Definitions 2.5.1, 2.5.4 and 2.5.10) and give some examples (Example 2.5.5). After that define locally constant sheaves (Definition 2.5.6) and prove Proposition 2.5.8. The main goal of this talk is to prove Theorem 2.5.9. For this you will need Construction 2.5.12 and Lemma 2.5.13. Conclude with Theorem 2.5.14.

**Talk 9: Riemann Surfaces (0).** (Chapter 3.1 and 3.2) Define the notion of a Riemann surface and of holomorphic maps and give examples (Chapter 3.1). State Proposition 3.2.1 (you have essentially seen a proof of this in complex analysis). Define the ramification index and note Corollary 3.2.3 and 3.2.4. Define the notion of a proper map and prove Proposition 3.2.6. After that discuss and try to explain the proof Theorem 3.2.7 which is the main goal of this talk. For this you need Lemma 3.2.8 and Proposition 3.2.9. For time reasons you may sketch some of the proofs or skip them completely.

**Talk 10: Relation with Field Theory (+).** (Chapter 3.3) Introduce the ring of meromorphic functions on a Riemann surface and prove Lemma 3.3.2. After that state Riemann's Existence Theorem and prove Proposition 3.3.5. For this you will need Lemma 3.3.6. After that state and discuss Theorem 3.3.7. You may focus on proving the essential surjectivity (Proposition 3.3.8) and only sketch (or skip) the fully faithfulness.

**Talk 11: The Absolute Galois Group of  $\mathbb{C}(T)$  (0).** (Chapter 3.4) Prove Theorem 3.4.1 and Lemma 3.4.2. Explain Remark 3.4.3 and deduce Corollary 3.4.4. After that define the notion of a free profinite group (Definition 3.4.6) and state Theorem 3.4.8. For this you need Proposition 3.4.9 which is purely group-theoretic. If time permits, try to explain the proof of this, otherwise just use it as a black-box.