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Group theory for physicists
Problem set 2 (for the exercise on Oct. 27)

Problem 1 Multiplication table of S_3

Construct the multiplication table of S_3.

Problem 2 Isotropy group

As in Sec. 1.6 of the lecture, let G be a group, M be a set, and

$$G_m = \{g \in G \mid gm = m\}$$

be the isotropy group of $m \in M$. Show that the set G_m is a group, i.e., that it satisfies the four group properties.

Problem 3 Orbits and isotropy groups

Show for the case of a finite group G that the following equation holds for a point m and its isotropy group G_m:

$$(\text{number of elements of the orbit of } m \text{ under } G) \cdot \text{order}(G_m) = \text{order}(G)$$

Problem 4 Equivalence classes

Prove the following statements from Sec. 1.7 of the lecture:

a) The identity forms a class by itself.

b) If the group is Abelian, each element forms a class by itself.

c) Every element of G belongs to one and only one class.

d) The order of the group is divisible by the number of elements in a class.
Hint: Use the result of Problem 3.