Universität Regensburg, Institut für Theoretische Physik
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WS 2024/25

Group theory for physicists
Problem set 10 (for the exercises in the week of Jan. 13)

Problem 1 Representations and characters of S_4

a) Draw all Young diagrams for S_4 and determine the dimensions of the corresponding irreps. Check that $\sum_i d_i^2 = 4!$ is satisfied.

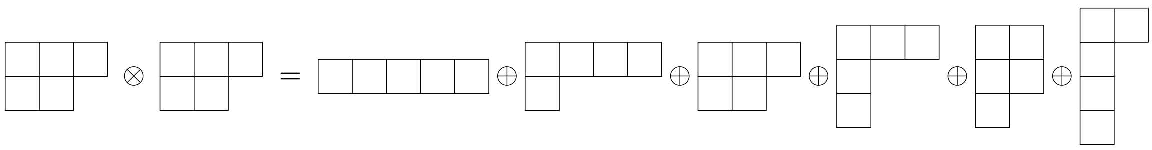
b) Compute the character table of S_4 and check the orthogonality relations for the characters. Hint: Some irreps are associated to each other.

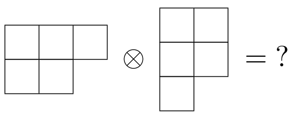
Problem 2 Representations and characters of S_5

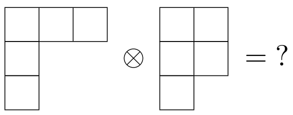
Same as problem 1, but now for S_5.

Problem 3 Product representations of S_5

Consider the following product representations of S_5 and find out which irreps of S_5 occur in them (including the corresponding multiplicities). Use the characters from problem 2. The result of part a) is given for your convenience, but you should still prove it.

a) 

b) 

c) 

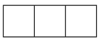
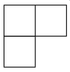
Check the dimensions on both sides of the equations.
Hint: Try to find an efficient way of computing the multiplicities a_k by regarding the character table as a matrix and expressing a_k as a scalar product involving a row of the character table.

Problem 4 Baryon decuplet and octet

In the quark model baryons consist of three quarks. Quarks have several quantum numbers, e.g., the z-component I_3 of isospin and the hypercharge $Y = B + S$, where B is the baryon number ($B = \frac{1}{3}$ for all quarks) and S is the strangeness. We consider the following three states: ψ_1 (up quark) with $(I_3, Y) = (\frac{1}{2}, \frac{1}{3})$, ψ_2 (down quark) with $(I_3, Y) = (-\frac{1}{2}, \frac{1}{3})$, and ψ_3 (strange quark) with $(I_3, Y) = (0, -\frac{2}{3})$. For a product wave function $\psi_i(1)\psi_j(2)\psi_k(3)$ involving three quarks, I_3 and Y are given by the sums of the I_3- and Y-values of the three quarks.

a) The functions $|ijk\rangle \equiv \psi_i(1)\psi_j(2)\psi_k(3)$ with $i, j, k = 1, \ldots, 3$ furnish a 27-dimensional representation of S_3. Find out which irreps of S_3 occur in this representation and the corresponding multiplicities. (The group action is given by $(12)|ijk\rangle = |jik\rangle$ etc.)

b) Find the 27 functions that transform in irreps of S_3 and span the irreducible subspaces corresponding to these irreps. Give the (I_3, Y)-values of these functions. Use the techniques from Sec. 4.2 of the lecture.

c) In an (I_3, Y)-diagram, draw points corresponding to the functions transforming in the irrep . In another (I_3, Y)-diagram, draw points corresponding to the functions transforming in the irrep .

d) Give a tentative physical interpretation of your results. We will discuss this topic in more detail in Sec. 8.7.4 of the lecture.