

**Group theory for physicists**  
**Problem set 2 (for the exercise on Oct. 28)**

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**Problem 1 Multiplication table of  $S_3$**

Construct the multiplication table of  $S_3$ .

**Problem 2 Isotropy group**

As in Sec. 1.6 of the lecture, let  $G$  be a group,  $M$  be a set, and

$$G_m = \{g \in G \mid gm = m\}$$

be the isotropy group of  $m \in M$ . Show that the set  $G_m$  is a group, i.e., that it satisfies the four group properties.

**Problem 3 Orbits and isotropy groups**

Show for the case of a finite group  $G$  that the following equation holds for a point  $m$  and its isotropy group  $G_m$ :

$$(\text{number of elements of the orbit of } m \text{ under } G) \cdot \text{order}(G_m) = \text{order}(G)$$

**Problem 4 Equivalence classes**

Prove the following statements from Sec. 1.7 of the lecture:

- a) The identity forms a class by itself.
- b) If the group is Abelian, each element forms a class by itself.
- c) Every element of  $G$  belongs to one and only one class.
- d) The order of the group is divisible by the number of elements in a class.  
Hint: Use the result of Problem 3.