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Group theory for physicists
Problem set 1 (for the exercises on Oct. 21)

Problem 1 Properties of the multiplication table

Show that in the multiplication table of a finite group each element appears once and only once in each row and each column.
Hint: Suppose that an element appears twice and show that this leads to a contradiction.

Problem 2 The group of order 3

Show that there is only one group of order 3 by constructing the multiplication table.

Problem 3 The rearrangement lemma

Consider a group with n different elements: $(I, a, b, c, \ldots, n)$. Show that in the ordered set of products $(aI, a^2, ab, ac, \ldots, an)$ the n different elements are reproduced in a new order.

Problem 4 Subgroups

Let H and K be two subgroups of a group G. Show that $H \cup K$ (i.e., the union of H and K) is a subgroup of G if and only if $H \subseteq K$ or $K \subseteq H$.