Universität Regensburg, Fakultät für Physik
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Übungen zur Quantenmechanik für LA und Nanoscience
Blatt 2 (für die Übungen in der Woche 30.10.-03.11.)

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1 Photoelectric effect and Compton scattering

a) The maximum energy of photoelectrons from aluminum is 2.3 eV for radiation of 2000 Å and 0.90 eV for radiation of 3130 Å. Use these data to calculate Planck's constant and the work function W required to remove an electron from aluminum.

b) An electron of energy 100 MeV collides with a photon of wavelength $3 \cdot 10^7$ Å (corresponding to the universal background of black-body radiation). What is the maximum energy loss suffered by the electron?

c) A beam of X-rays is scattered off electrons at rest. What is the energy of the X-rays if the wavelength of the X-rays scattered at 60° relative to the beam axis is 0.035 Å?

2 Black-body radiation

a) Recover Rayleigh-Jeans' and Wien's law as limiting cases of Planck's law.

b) Derive the Stefan-Boltzmann law from Planck's law, i.e., show that the spectrally integrated power per unit area radiated by a black body at temperature T is given by

$$S = \frac{c}{4} \int_0^\infty d\nu \, u(\nu, T) = \sigma T^4$$

with a constant σ. Determine σ by explicit integration.

3 Wave packets

Localized quantum-mechanical objects may be represented by wave packets. In technical terms, these are linear superpositions of plane waves, which individually would be the prototype of a maximally delocalized object. Since the wave functions of quantum mechanics are complex, one uses complex plane waves e^{ikx}. In the one-dimensional case a wave packet may be written as

$$f(x) = \int_{-\infty}^{+\infty} dk \, g(k) e^{ikx} \, .$$

For suitable $g(k)$, $f(x)$ will be localized and may be interpreted as a particle whose size corresponds to the localization length.

a) For $g(k) = e^{-\alpha(k-k_0)^2}$, determine $f(x)$ and then $|f(x)|^2$. The latter is proportional to the probability density of finding the particle at position x. Give an interpretation of the real and positive constant α.
Hint: When computing $f(x)$, rearrange the terms in the exponent in order to get a square plus a constant and then use the well-known relation

$$\int_{-\infty}^{+\infty} dk \, e^{-\alpha k^2} = \sqrt{\frac{\pi}{\alpha}} \, .$$

Are there any subtleties when using this relation here?

b) We now replace this wave packet by

$$f(x,t) = \int_{-\infty}^{+\infty} dk\, g(k) e^{ik(x-ct)} \,.$$

What is the physical difference compared to the previous situation?

c) A further generalization is given by

$$f(x,t) = \int_{-\infty}^{+\infty} dk\, g(k) e^{i(kx-\omega(k)t)} \,,$$

where we again assume $g(k) = e^{-\alpha(k-k_0)^2}$. For large α the exponent in the integral may be expanded to good accuracy about k_0. Perform this expansion up to second order in $k - k_0$ and again compute $f(x,t)$ and $|f(x,t)|^2$. What can one conclude about the behavior of the wave packet?