University of Regensburg WS 2019/20

Quantum Information Theory

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Tue. H33 13pm c.t. & **Thu.** H34, 3pm c.t.
Mon. 12pm c.t., H33

Sheet 7

1 Remainder .... [8P]

(a) Which remainder is left when 3^{2049} is divided by 68?

(b) Find a such that $2741^{897} \equiv a \pmod{3277}$.

2 Continued Fraction .... [18P]

(a)[2P] Write 3.14159 as a simple continued fraction (SCF).

(b)[2P] Write 31/123 into a SCF with an even and odd number of terms.

(c)[2P] Given the approximation 3.14159 to π, calculate its successive convergents.

(d)[3P] Show that $[2, \overline{2,4}] = \sqrt{6}$.

(e)[3P] Given $x = [\overline{b,a}]$ where b is a multiple of a, show that x satisfies the equation $x^2 - bx - b/a = 0$.

(f)[6P] Let p_i be the numerator and q_i the denominator of the ith convergent c_i of the continued fraction $[a_1, a_2, \ldots, a_n]$. Prove that

$$\frac{p_n}{p_{n-1}} = [a_n, a_{n-1}, \ldots, a_1], \quad \text{and} \quad \frac{q_n}{q_{n-1}} = [a_n, a_{n-1}, \ldots, a_2]\,. \tag{1}$$

3 Circulant Matrix .... [9P]

Once again we take a look at the Quantum Fourier Transformation. Here, we would like to study the operator F with $F_{kj} = \omega^{jk}/\sqrt{N}$ (see lecture notes) in more detail.
Consider a $N \times N$ *circulant matrix* C. It is defined by

$$C := \begin{pmatrix} c_0 & c_1 & c_2 & \cdots & c_{N-1} \\ c_{N-1} & c_0 & c_1 & \cdots & c_{N-2} \\ c_{N-2} & c_{N-1} & c_0 & \cdots & c_{N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_1 & c_2 & c_3 & \cdots & c_0 \end{pmatrix}. \tag{2}$$

A circulant matrix with $c_i = \delta_{1i}$, denoted by C_p, is called the *primary permutation matrix*. Furthermore, let us define $f(\lambda) = \sum_{i=0}^{N-1} c_i \lambda^i$.

(a) Show that $C = f(C_p)$ and that C is normal (what is the consequence of the latter?).

(b) Given the Nth primitive root of unity, denoted by ω, show that the eigenvalues of C are $f(\omega^m)$, $m = 0, 1, \ldots, N-1$.

(c) Show that F with $F_{jk} = \omega^{(j-1)(k-1)}/\sqrt{N}$, $j, k = 1, \ldots, N$ diagonalizes C.