Quantum Information Theory

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Tue. H33 13pm c.t. & **Thu.** H34, 3pm c.t.
Mon. 12pm c.t., H33

Sheet 12

1 Depolarizing Channel .... [4P]

Given a single qubit quantum operations model

$$\mathcal{E}_p(\rho) = \frac{p}{2}\mathbb{1} + (1-p)\rho \quad \text{with} \quad 0 \leq p \leq 1\ . \tag{1}$$

(a) Write down $\mathcal{E}_p(\rho)$ in the operator-sum representation.
Hint: Recall sheet 10, Ex. 4.

(b) Rewriting the density operator in the Bloch sphere representation $\rho = (\mathbb{1} + \mathbf{r}\cdot\boldsymbol{\sigma})$, $\mathbf{r} \in \mathbb{R}^3$, gives us a better physical insight. What happens to $\mathbf{r}$ if $\mathcal{E}_p$ is applied?

2 Rotations .... [8P]

Consider the following set $\mathcal{G}$ of operations in $\mathbb{R}^3$,[1]

E	identity,
C_{2x}, C_{2y}, C_{2z}	proper rotations through π about the x-, y- and z-axis resp.,
C_{4y}, C_{4y}^{-1}	proper rotations through $\pi/2$ about the y-axis,(C_{4y} right hand screw sense),
C_{2c}, C_{2d}	proper rotations through π about the c- and d-axis resp.,

with the c-axis pointing in the $(\mathbf{e}_x + \mathbf{e}_z)$ direction and the d-axis pointing in the $(-\mathbf{e}_x + \mathbf{e}_z)$ direction.

(a) Assign to each of the above operations R_i a non-singular 3×3 matrix $\Gamma(R_i)$ such that $\Gamma(R_iR_j) = \Gamma(R_i)\Gamma(R_j)$. Show that the set of R_i represents a group of order 8.
Hint: You can use a computing sys. like Mathematica, Matlab, Maple etc. to simplify your life.

(b) Find all subgroups and their order.
Hint: Use Lagrange's theorem.

(c) Let $\mathcal{S} = \{E, C_{2y}\}$. Write down the multiplication table of the factor group $\mathcal{G}/\mathcal{S}$.

(d) Find a homomorphic mapping of $\mathcal{G}$ onto $\mathbb{Z}_2 = \{-1, 1\}$.

3 Swap operator for continuous variables .... [*10P]

Quantum information theory can also make use of continuous variables (infinite dimensional Hilbert space). Using the later can allow for algorithms with higher efficiencies in practical applications. An example is the application of multi-photon fields of light over single-photon systems. To get a small insight, we consider the

[1] A rotation R is *proper* if $\det(R) = +1$.

shift operator in quantum optics which is the *displacement operator* $D(\alpha)$. Recall from the QM lecture that this operator acts on the vacuum state by displacing it into a coherent state,

$$D(\alpha)\left|0\right\rangle = \exp\left(\alpha a^\dagger - \alpha^* a\right) = \left|\alpha\right\rangle, \quad \text{with} \quad \alpha \in \mathbb{C}\,, \tag{2}$$

where $a(a^\dagger)$ is a Bose annihilation(creation) operator. In the following we would like to apply quantum operations on states which are now coherent states.
Let $|\gamma_1\rangle$, $|\gamma_2\rangle$ be two coherent states. How does the swap operator U_{swap} with

$$U_{\text{swap}}(|\gamma_1\rangle \otimes |\gamma_2\rangle) = |\gamma_2\rangle \otimes |\gamma_1\rangle \tag{3}$$

look like?
Hint: Start with $U(\alpha) = \exp\left(\alpha a_1^\dagger a_2 - \alpha^* a_1 a_2^\dagger\right)$.