University of Regensburg WS 2019/20

Quantum Information Theory

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Tue. H33 13pm c.t. & **Thu.** H34, 3pm c.t.
Mon. 12pm c.t., H33

Sheet 1

1 Guessing States .... [6P]

Alice prepares one of the following states,

$$|\psi_1\rangle = |0\rangle \quad \text{or} \quad |\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \tag{1}$$

and gives it to Bob. Bob has to identify the state using three *positive* operators Γ_1, Γ_2 and Γ_3 with $\sum_i \Gamma_i = \mathbb{1}$ under the following conditions:

- He is allowed to say he does not know the state.
- Sometimes his measurement has to give the correct answer.
- He is never allowed to give a wrong answer.

Given $\Gamma_1 = (\sqrt{2}/(1+\sqrt{2}))\,|1\rangle\langle 1|$, what are the other two states? *Beware: The operators Γ_i have to be positive!*

2 Hadamard operator I .... [6P]

Calculate the eigenvectors of the Hadamard operator $H^{\otimes 2}$. To do so, write $H^{\otimes 2}$ in the Bell basis.

3 Density Matrix I .... [6P]

(a) Let Λ_1 be an ensamble of states $\{|0\rangle\,,\ |1\rangle\}$ which constitute a density matrix

$$\rho = \alpha^2\,|0\rangle\langle 0| + \beta^2\,|1\rangle\langle 1| \quad \text{with} \quad \alpha^2 + \beta^2 = 1\,. \tag{2}$$

How does a general ensamble of states $\{|a\rangle\,,\ |b\rangle\}$ look like which yields the same density matrix with

$$\rho = \frac{1}{2}\,|a\rangle\langle a| + \frac{1}{2}\,|b\rangle\langle b|\ ? \tag{3}$$

(b) Given an arbitrary mixed state qubit, show that its density matrix can be written as

$$\rho = \frac{\mathbb{1} + \mathbf{r}\cdot\boldsymbol{\sigma}}{2} \tag{4}$$

with the Pauli vector $\boldsymbol{\sigma}$ and the Bloch vector $\mathbf{r}$, $\|\mathbf{r}\| \leq 1$.

4 Schmidt Decomposition . . . . [6P]

Find the Schmidt decomposition of the following states which are consisting of two qubits,

(a) $\dfrac{|00\rangle + |11\rangle}{\sqrt{2}}$,

(b) $\dfrac{|00\rangle + |01\rangle + |10\rangle + |11\rangle}{2}$,

(c) $\dfrac{|00\rangle + |01\rangle + |11\rangle}{\sqrt{3}}$.