University of Regensburg WS 2019/20

Quantum Information Theory

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Tue. H33 13pm c.t. & **Thu.** H34, 3pm c.t.
Mon. 12pm c.t., H33

Sheet 0

1 The NAND Gate . . . . [6P]

The classical NAND gate is defined by $x \,\mathrm{NAND}\, y := \overline{x \wedge y}$. Replace the

(a) OR gate [x OR $y := x \vee y$],

(b) AND gate [x AND $y := x \wedge y$],

(c) XNOR gate [x XNOR $y := \overline{x \veebar y} \equiv \overline{x \oplus y}$]

with a combination of NAND gates by applying Boolean algebra. Try to use a minimal number of NAND gates!

2 Stern Gerlach Experiment . . . . [3P]

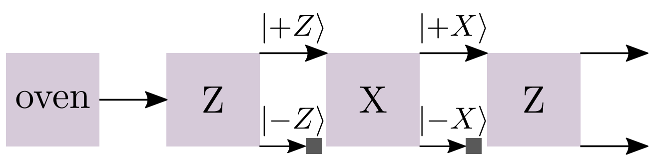

Figure 1: Measurement

An oven is ejecting particles with spin 1/2. According to Fig. 1 we first measure the spin according to the eigenbasis $\{|\pm Z\rangle\}$ of the Pauli matrix σ_z. The device blocks the spin-down component. In the next part of the device the measurement is carried out by measuring in the σ_x basis with blocking of $|-X\rangle$. Finally, we measure again in the σ_z basis. What are the probability densities of the last both channels?

3 Tensor Product . . . . [7P]

(a) [2P] Let $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, $|-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$. Write out

(i) $|+\rangle^{\otimes 2}$

(ii) $|-\rangle^{\otimes 3}$

explicitly as a tensor product using $|.\rangle\,|.\rangle$ and as a Kronecker product.

(b) [2P] Show that the tensor product of two projection operators is a projection operator.

(c) [3P] What is the expectation value of the $\sigma_x \otimes \sigma_z$ operator in the $|\psi\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ state where $\{|0\rangle, |1\rangle\}$ are the single particle eigenstates of σ_z?

4 Unitary Transformation . . . . [6P]

Show that an arbitrary 2×2 unitary matrix can be written as

$$U = e^{i\alpha}\Lambda(\beta)R(\gamma)\Lambda(\nu) \tag{1}$$

with

$$\Lambda(\nu) = \begin{pmatrix} e^{i\nu} & 0 \\ 0 & 1 \end{pmatrix}, \quad R(\gamma) = \begin{pmatrix} \cos(\gamma) & \sin(\gamma) \\ -\sin(\gamma) & \cos(\gamma) \end{pmatrix} \quad \text{with} \quad \alpha, \beta, \gamma, \nu \in \mathbb{R}\,. \tag{2}$$

5 Entanglement . . . . [8P]

Given the state $|\psi\rangle$ from Ex. 3 (c), can you find a state $|a\ b\rangle \equiv |a\rangle \otimes |b\rangle$ with $|\psi\rangle = |ab\rangle$?