From Amenable to Inner-amenable

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Ventotene, September 11th, 2023

Question

Let G be a residually finite group with residual chain $(G_i)_{i\in\mathbb{N}}$, $k \in \mathbb{N}$, K be a field. When is

$$\lim_{i\to\infty} \frac{\dim_K H_k(BG_i; K)}{[G : G_i]} = 0?$$

[1] +some finiteness conditions

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For example if G is amenable and infinite[1]

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Observation

Also true for $G = A \times \Gamma$ where

- A: infinite and amenable
- Γ: arbitrary group

[1] +some finiteness conditions

Definition (amenability)

A group G is *amenable* if there exists a left-invariant mean $\ell^\infty(G, \mathbb{R}) \to \mathbb{R}$.

Definition (inner-amenability)

A group G is *inner-amenable* if there exists a conjugation-invariant[2] mean $\ell^\infty(G, \mathbb{R}) \to \mathbb{R}$.

[2] and atomless

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Example

- Infinite, amenable groups
- $A \times \Gamma$, where A: infinite amenable
- $\mathrm{BS}(m, n)$
- Not: F_2.

[2] and atomless

Theorem ([Usc22, Corollary 1.3])

Let G be a torsion-free, inner-amenable[3] *group. Then,*

$$\lim_{i\to\infty} \frac{\dim_K H_1(BG_i;K)}{[G:G_i]} = 0,$$

for any residual chain $(G_i)_{i\in\mathbb{N}}$ and field K.

[3] and finitely generated, residually finite

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Extend results from amenable to inner-amenable groups!

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References

Matthias Uschold.
Torsion homology growth and cheap rebuilding of inner-amenable groups.
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