From Amenable to Inner-amenable

Matthias Uschold, University of Regensburg

Göttingen, October 9th or 10th, 2023

Question

Let G be a residually finite group with residual chain $(G_i)_{i\in\mathbb{N}}$, $k \in \mathbb{N}$, K be a field. When is

$$\lim_{i\to\infty} \frac{\dim_K H_k(BG_i; K)}{[G : G_i]} = 0?$$

[1] +some finiteness conditions

Question

Let G be a residually finite group with residual chain $(G_i)_{i\in\mathbb{N}}$, $k \in \mathbb{N}$, K be a field. When is

$$\lim_{i\to\infty} \frac{\dim_K H_k(BG_i; K)}{[G : G_i]} = 0?$$

Answer

For example if G is amenable and infinite[1]

[1] +some finiteness conditions

Question

Let G be a residually finite group with residual chain $(G_i)_{i\in\mathbb{N}}$, $k \in \mathbb{N}$, K be a field. When is

$$\lim_{i\to\infty} \frac{\dim_K H_k(BG_i; K)}{[G : G_i]} = 0?$$

Answer

For example if G is amenable and infinite[1]

Observation

Also true for $G = A \times \Gamma$ where

- A: infinite and amenable
- Γ: arbitrary group

[1] +some finiteness conditions

Definition (amenability)

A group G is *amenable* if there exists a left-invariant mean $\ell^\infty(G, \mathbb{R}) \to \mathbb{R}$.

Definition (inner-amenability)

A group G is *inner-amenable* if there exists a conjugation-invariant[2] mean $\ell^\infty(G, \mathbb{R}) \to \mathbb{R}$.

[2] and atomless

Definition (inner-amenability)

A group G is *inner-amenable* if there exists a conjugation-invariant[2] mean $\ell^\infty(G, \mathbb{R}) \to \mathbb{R}$.

Example

- Infinite, amenable groups
- $A \times \Gamma$, where A: infinite amenable
- $\mathrm{BS}(m, n)$
- PL-homeomorphisms of $\mathbb{R}$ (such as Thompson's group F)
- Not: F_2.

[2] and atomless

Theorem ([Usc22, Corollary 1.3])

Let G be a torsion-free, inner-amenable[3] *group. Then,*

$$\lim_{i\to\infty} \frac{\dim_K H_1(BG_i; K)}{[G : G_i]} = 0,$$

for any residual chain $(G_i)_{i\in\mathbb{N}}$ and field K.

[3] and finitely generated, residually finite

Theorem ([Usc22, Corollary 1.3])

Let G be a torsion-free, inner-amenable[3] *group. Then,*

$$\lim_{i\to\infty} \frac{\dim_K H_1(BG_i; K)}{[G : G_i]} = 0,$$

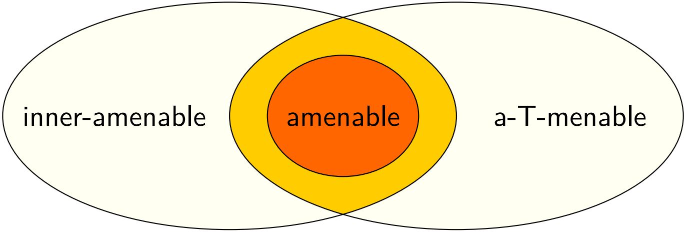
for any residual chain $(G_i)_{i\in\mathbb{N}}$ and field K.

Challenge

Extend results from amenable to inner-amenable groups!

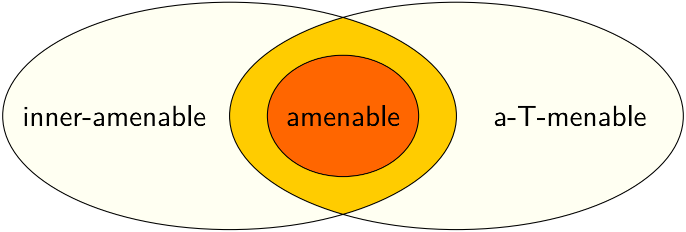
[3] and finitely generated, residually finite

Bonus slide: Relation to a-T-menable



[BCS00]

Bonus slide: Relation to a-T-menable



[BCS00]

Thanks!

References

Cédric Béguin and Tullio Ceccherini-Silberstein.
Weak forms of amenability for one-relator groups.
Bull. Belg. Math. Soc. - Simon Stevin, 7(1):135–148, 2000.

Matthias Uschold.
Torsion homology growth and cheap rebuilding of inner-amenable groups.
arXiv:2212.07916, 2022.