Exercise sheet 9

Algebraic Geometry I
Winter term 2017/2018

Exercise 1
Let $k$ be a field. Describe the scheme

$$\text{Spec}(\text{Quot}(k[X])) \times_{\text{Spec}(k)} \text{Spec}(\text{Quot}(k[Y])).$$

Exercise 2
Consider a scheme $S$, a scheme $X \in \text{Sch}/S$ and the fibre product $X \times_S X$. The universal property induces a morphism

$$\Delta: X \to X \times_S X$$

of schemes. On topological spaces, this yields a subset

$$\Delta(X) \subseteq \{y \in X \times_S X \mid \text{pr}_1(y) = \text{pr}_2(y)\}.$$

Show that this need not be an equality.

Hint: A candidate for $X$ can be obtained by gluing two affine lines $\mathbb{A}^1$ along the open subsets $\mathbb{A}^1 \setminus \{0\}$. 