Exercise sheet 5

Algebraic Geometry I
Winter term 2017/2018

Exercise 1
Let $X$ be a topological space and $\mathcal{F}: \text{Ouv}(X)^{\text{op}} \to \text{Set}$ a sheaf. Let $s, t \in \mathcal{F}(X)$ and show that

$$Y := \{x \in X \mid s_x = t_x\}$$

is an open subset of $X$.

Exercise 2
Consider the presheaf

$$\mathcal{C}_{bd}(-, \mathbb{R}): \text{Ouv}(\mathbb{R})^{\text{op}} \to \text{Set} \quad U \mapsto \{f: U \to \mathbb{R} \mid f \text{ is continuous and bounded}\}$$

and show that it is not a sheaf.

Exercise 3
Let $\pi: \tilde{X} \to X$ be a continuous map of topological spaces. Show that the presheaf

$$\mathcal{S}\text{ec}(\pi): \text{Ouv}(X)^{\text{op}} \to \text{Set} \quad U \mapsto \{s: U \to \tilde{X} \mid \pi s = \text{id}_U \text{ and } s \text{ is continuous}\}$$

is a sheaf. Consider the covering space $\pi: \mathbb{R} \to S^1$ given by the exponential function and draw a picture. Show that $\mathcal{S}\text{ec}(\pi)$ is not a constant sheaf (i.e. not the sheafification of a constant presheaf).

Exercise 4
Let $f, g: \mathcal{F} \to \mathcal{G}$ be maps of presheaves from a presheaf $\mathcal{F}$ into a sheaf $\mathcal{G}$ which coincide on each stalk, i.e. $f_x = g_x$ for all $x \in X$. Show that $f = g$. 