EXERCISE 1
Consider the topological space $X := \text{Spec}(A)$ for a ring $A$.

1. Let $\{Z_\alpha\}_\alpha$ be a collection of closed subsets $Z_\alpha \subseteq X$ such that the intersection of each finite subcollection is non-empty. Show that the whole intersection $\bigcap_\alpha Z_\alpha$ is non-empty.

2. Show exercise (a) in the proof of Corollary 5.10 of the script.

EXERCISE 2
Decide whether the canonical morphism $\coprod_{N} \text{Spec}(\mathbb{F}_2) \to \text{Spec}(\prod_{N} \mathbb{F}_2)$ is a homeomorphism in the Zariski topology.

EXERCISE 3
Show that $\mathbb{C}^2$ is not homeomorphic to $\mathbb{C}^1 \times \mathbb{C}^1$ in the Zariski topology.

EXERCISE 4
Let $k$ be a field.

1. Show that $\text{Spec}(k[X_1, \ldots, X_n])$ is infinite if $n \geq 1$.

2. Let $A$ be a $k$-algebra of finite type with $\text{Spec}(A)$ finite. Conclude that $A$ is finite over $k$. (You may use that $\text{Spec}(f)$ is surjective if $f$ is injective and integral.)