Exercise sheet 12
Algebraic Geometry I
Winter term 2017/2018

EXERCISE 1
Consider a morphism \( f : X \to Y \) of schemes. Show that there is a unique closed subscheme \( i : Z \to Y \), called the \textit{scheme theoretic image of} \( f \), with the following universal property:

1. There is a factorization \( X \to Z \xrightarrow{i} Y \) of \( f \).
2. For every other such factorization \( X \to Z' \xrightarrow{i'} Y \) of \( f \) with \( i \) a closed immersion, there exists a unique map \( t : Z \to Z' \) making everything commute.

Show moreover that, if \( X \) was a reduced scheme, \( Z \) is the closure of the image \( \overline{f(X)} \subseteq Y \) together with the reduced closed subscheme structure. (Hint: Consider the case of an affine affine \( Y \) first.)

EXERCISE 2
Show that there cannot be an equivalence of categories between the category \( \textbf{Set} \) of sets and its opposite category \( \textbf{Set}^{op} \).
(Hint: Use coproducts, products, final and initial objects.)