Exercise sheet 10

Algebraic Geometry I
Winter term 2017/2018

For the first two exercises, let $k = \overline{k}$ be an algebraically closed field and consider the morphism

$$f : X := \text{Spec}(k[X, Y]/(X - Y^2)) \to S := \text{Spec}(k[T])$$

of schemes induced by $T \mapsto X$ (draw a picture!).

**EXERCISE 1**
Calculate the fibers $X_s$ of $f$ at all points $s := (T - a) \in S$ for $a \in k$.

**EXERCISE 2**
Calculate the fiber $X_\eta$ of $f$ at the generic point $\eta \in S$.

**EXERCISE 3**
Fix positive integers $a, b$ and $c$ and consider the Fermat scheme

$$S := \text{Spec}(\mathbb{Z}[X, Y, Z]/(X^a + Y^b - Z^c))$$

with open subscheme $U := S \setminus V((X, Y, Z) + (X^a + Y^b - Z^c))$. Show that the set of morphisms

$$\text{Hom}_{\text{Sch}}(\text{Spec}(\mathbb{Z}), U)$$

is in bijection with the integer solutions $(x, y, z)$ to $X^a + Y^b = Z^c$ with $\gcd(x, y, z) = 1$. 