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A generalized model of non-thermal noise in the electromagnetic environment of small-capacitance tunnel junctions

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Abstract. – We present a generalization of the existing theory for tunneling in the presence of an electromagnetic environment, which applies to arbitrary networks of linear elements, with each impedance at its individual equilibrium temperature. The fluctuations of such an environment are non-thermal. As an example we discuss the effect of high-temperature fluctuations in the environment. We show that in general these fluctuations exhibit characteristics of Gaussian-correlated and white noise.

In order to observe charging effects in small-capacitance tunnel junctions (see, *e.g.*, ref. [1]) it is necessary to connect the single-electron device to the external world, *i.e.* sources, measuring devices etc. The interaction of the tunneling electrons with the environment is crucial for the behavior of tunnel-junction systems; it is described by a well-developed theory [2]-[5] in which the electromagnetic environment is modeled by an impedance $Z(\omega)$. According to this theory, at finite temperatures the environment can also *activate* tunneling [6]. Martinis and Nahum [7] first pointed out that parts of the environment at considerably higher temperatures than the electron temperature T_{el} in the junction act as noise sources which may cause significant deviations from the expected device operation (see, *e.g.*, refs. [7]-[10]). They considered a tunnel junction in a circuit with a single impedance whose temperature is much higher than T_{el}; the expressions they obtained for the tunneling rate adequately describe the results of various experiments.

In an experiment the environment is a complicated network where many temperatures are present. It is, therefore, desirable to develop a theory for a more general class of circuits (see also ref. [9]). In this letter we derive an expression for the tunneling rate which is valid for arbitrarily high impedances and arbitrary noise power, whereas in earlier calculations such results were obtained perturbatively. We present in a convenient form the expression for quantum non-equilibrium noise in linear networks which have not been discussed in this context before. This expression is valid for arbitrarily complicated non-equilibrium networks which may be found in experiments; in particular it is easily generalized for a distribution of resistances or temperatures. We start with a brief review of the standard approach. An interpretation which is slightly different from the usual one leads us straightforwardly to the generalization of the existing theory. We then consider a simple application of our model for a quite general type of frequency dependence of the impedance $Z(\omega)$. We find that high-temperature fluctuations in the environment are characterized by a coexistence of Gaussian (*i.e.* the correlation function is Gaussian) and white noise.

Influence of the electromagnetic environment on single-electron tunneling rates. – In a general (multi-junction) circuit the effect of the environment on single-electron tunneling in a certain junction is accounted for by considering a suitable external impedance which includes the circuit and the other junctions, seen as capacitors [5]. Therefore, it is sufficient to consider a single junction (with capacitance C) in series with an impedance $Z(\omega)$ driven by a constant voltage source V_0. The tunneling rate Γ_{env} is given by [5]

$$\Gamma_{\mathrm{env}}(eV_0) = \int_{-\infty}^{+\infty} \mathrm{d}E\ \Gamma(eV_0 - E)\ P(E)\,, \tag{1}$$

$$P(E) = 1/(2\pi\hbar) \int_{-\infty}^{+\infty} \mathrm{d}t \exp\left[J(t) + (iEt/\hbar)\right]. \tag{2}$$

Here Γ is the tunneling rate for $Z = 0$; moreover, we have defined

$$e^{J(t)} \equiv \langle e^{i\tilde{\varphi}(t)} e^{-i\tilde{\varphi}(0)} \rangle\,. \tag{3}$$

The phase operator $\tilde{\varphi}$ is related to the voltage across the tunnel junction $V(t)$ via $\tilde{\varphi}(t) = (e/\hbar)\int_{-\infty}^{t} \mathrm{d}t\ [V(t) - V_0]$. Further, $\langle \ldots \rangle$ denotes the average with respect to the density matrix of the environment. By applying Wick's theorem it can be shown [5] that for a linear external impedance, $J(t)$ is directly related to the phase-phase correlation function $J(t) = \langle [\tilde{\varphi}(t) - \tilde{\varphi}(0)]\tilde{\varphi}(0)\rangle$. Due to the fluctuation-dissipation theorem we have

$$\langle \tilde{\varphi}\tilde{\varphi}\rangle_\omega = \frac{1}{\omega^2}\langle (V - V_0)(V - V_0)\rangle_\omega = \frac{1}{\omega^2}\frac{2\hbar\omega}{1 - e^{-\beta_{\mathrm{env}}\hbar\omega}}\mathrm{Re}Z_{\mathrm{t}}(\omega)\,. \tag{4}$$

Here we introduced the temperature of the environment $T_{\mathrm{env}} = \beta_{\mathrm{env}}^{-1}/k_{\mathrm{B}}$ and the total impedance $Z_{\mathrm{t}}(\omega) = 1/[i\omega C + Z^{-1}(\omega)]$ which is seen by the tunnel junction. According to eq. (4), the dissipative part of the impedance causes thermal as well as quantum fluctuations of the phase which are experienced by the tunneling electron and, thus, affect the tunneling rate. We can re-interpret this result recalling the well-known fact that a fluctuating impedance can be represented by an ideal non-fluctuating impedance and a fluctuating voltage source $\tilde{U}$ in series (cf. fig. 1 *a*)). The spectral power of the voltage fluctuations at the capacitor C due to the voltage $\tilde{U}$ is easily seen to be (cf. fig. 1 *b*))

$$\langle (V - V_0)(V - V_0)\rangle_\omega = \frac{1}{|1 + i\omega CZ|^2}\langle \tilde{U}\tilde{U}\rangle_\omega = \mathrm{Re}Z_{\mathrm{t}}(\omega)\frac{2\hbar\omega}{1 - e^{-\beta_{\mathrm{env}}\hbar\omega}}\,. \tag{5}$$

Comparing this result with eq. (4) we can say that the phase fluctuations at the junction are produced by the fluctuating voltage source $\tilde{U}$.

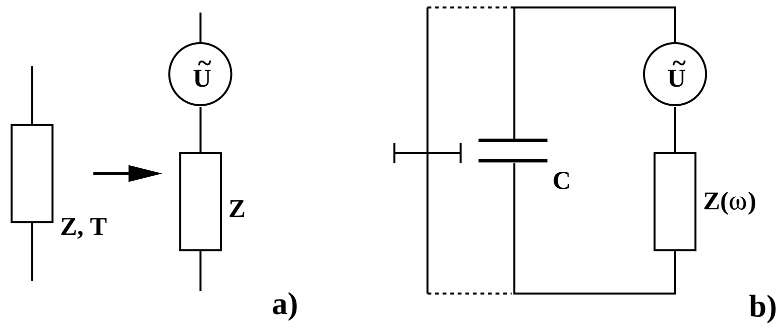

Fig. 1. – *a*) Noisy resistance, represented by a series noise voltage generator. *b*) Illustration of the origin of the total impedance Z_t in eq. (5).

Generalization for an arbitrary network. – The formalism presented above is valid for an environment in thermal equilibrium. In order to generalize it to non-equilibrium situations, the correlation function in eq. (3) has to be evaluated where the average no longer denotes the trace over an equilibrium density matrix. The general case of an arbitrary density matrix (*e.g.*, for a generic situation far from equilibrium or a circuit containing a non-linear element) cannot be treated since Wick's theorem does not apply. Therefore, we assume: i) the environment consists of parts which can be represented by huge sets of harmonic oscillators (*e.g.*, a network of linear passive elements); ii) each part is in local equilibrium at its own temperature; iii) different parts produce uncorrelated noise. In this case it is possible to generalize the equilibrium method. The problem then reduces to the evaluation of the phase-phase correlation function $J(t)$ due to the impedances $Z_n(\omega)$ at the temperature T_n, $n = 1, \ldots, N$ by using network theory. The above assumptions are appropriate in many experimental situations.

The phase fluctuations in the presence of the generalized environment arise due to the noise sources $\tilde{U_n}$ of each impedance as discussed before. The total fluctuating voltage at the junction is now given by $V = \sum_{n=1}^{N} \kappa_n(\omega)\tilde{U}_n(\omega)$, where the complex parameters $\kappa_n(\omega)$ have to be determined by applying Kirchhoff's laws in the given network. Thus, we obtain the voltage-voltage correlator at the junction for an arbitrary network

$$\langle (V - V_0)(V - V_0)\rangle_\omega = \sum_{n=1}^{N} |\kappa_n(\omega)|^2 \mathrm{Re} Z_n(\omega) \frac{2\hbar\omega}{1 - e^{-\beta_n \hbar\omega}} \,, \qquad (6)$$

where we have taken into account the voltage $V_0 = \sum \kappa_n(0) V_{0n}$ at the junction which is produced by constant sources V_{0n} in the circuit. Given the voltage correlator, it is straightforward to determine the expressions for $J(t)$ and $P(E)$ by using eqs. (2)-(4).

The junction sees the network as some total impedance $Z_\mathrm{t}(\omega)$ which obeys the relation $\mathrm{Re}Z_\mathrm{t}(\omega) = \sum |\kappa_n(\omega)|^2 \mathrm{Re}Z_n(\omega)$. It guarantees that for equal temperatures $T_1 = \ldots = T_N$ the result (5) for a single impedance is correctly reproduced. We can prove this relation by noting that for each impedance Z_n the system represents a linear two-port with the junction at one and the sub-circuit of the impedance Z_n at the other one-port. From the reciprocity theorem it follows that a current source I at the port of the junction produces a current $I_n = -\kappa_n I$ in the sub-circuit of Z_n. Considering the power dissipated in the network we find $|I|^2 \mathrm{Re}Z_\mathrm{t} = \sum |I_n|^2 \mathrm{Re}Z_n = \sum |I|^2 |\kappa_n|^2 \mathrm{Re}Z_n$ and, thus, the relation for $\mathrm{Re}\, Z_\mathrm{t}$.

The low-impedance environment. – First, we calculate an analytic expression for $P(E)$ if there is one small Ohmic impedance $\mathrm{Re}Z(\omega) \ll R_K = h/e^2$ at some temperature T_env in the

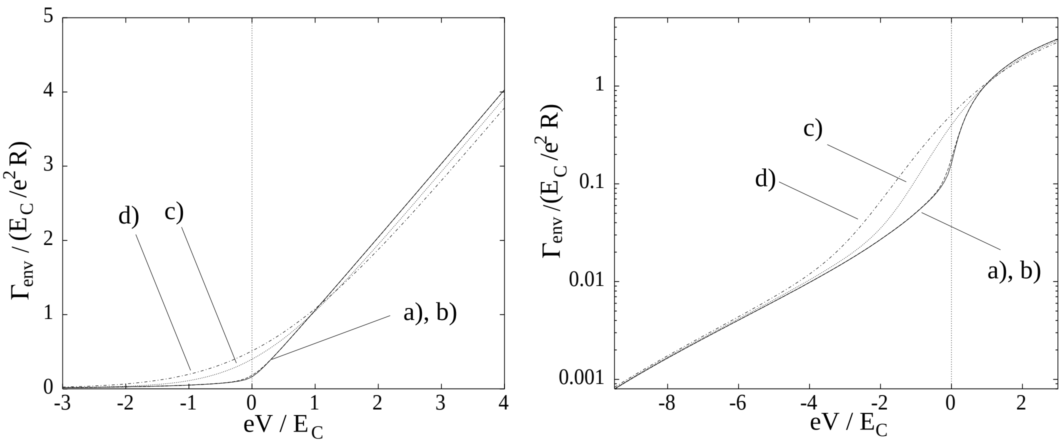

Fig. 2. – Tunneling rate $\Gamma_{\rm env}$ in a normal junction in linear and logarithmic representation for different ratios $\alpha_{\rm l}/\alpha_{\rm h}$: *a*) 10.0, *b*) 1.0, *c*) 0.15, *d*) 0.1.

circuit. To save writing we define $R \equiv {\rm Re}Z(\omega)$. While $|{\rm Im}J(t)| \ll 1$ for all times we find for the real part

$$\begin{cases} |{\rm Re}J(t)| \ll 1\,, & |t| < \hbar\beta_{\rm env}/2\,, \\ {\rm Re}J(t) \;\simeq\; -(2\pi R/R_K)|t|/(\hbar\beta_{\rm env})\,, & |t| > \hbar\beta_{\rm env}/2\,. \end{cases} \tag{7}$$

We note that $J(t) \propto |t|$ for large times, *i.e.* it is not possible to approximate $\exp[J(t)]$ by expanding the exponential function to first power in $J(t)$. By inserting the large-$|t|$ behavior of $J(t)$ into eq. (2), we find $P(E)$ for small energies:

$$P(E) = (\hbar\alpha/\pi)/[(\hbar\alpha)^2 + E^2]\ , \qquad \alpha = (2\pi R/R_K)/(\hbar\beta_{\rm env})\ , \qquad |E| < \hbar\alpha\,. \tag{8}$$

Integrating by parts eq. (2) and inserting eq. (7), we obtain the result

$$P(E) = \frac{1}{\pi E}\int_{-\infty}^{+\infty}{\rm d}t\int_{-\infty}^{+\infty}{\rm d}\omega\frac{R}{R_K}\frac{e^{-i(\omega-(E/\hbar))t+J(t)}}{1-e^{-\beta_{\rm env}\hbar\omega}} = \frac{1}{E}\frac{2R}{R_K}\frac{1}{1-e^{-\beta_{\rm env}E}}\,, \qquad |E| > \hbar\alpha\,, \tag{9}$$

for large energies, which coincides with the one in ref. [7].

Coexistence of Gaussian and white noise. – We now consider a tunnel junction in an environment at a temperature $T_{\rm env}$ which is of the order of the charging energy of the junction. For convenience we will attribute the full frequency dependence of the impedance to the parameters $\kappa_n(\omega)$. Let us assume an environment which has a high impedance at frequencies below a cut-off frequency ω_0 and low-impedance properties for larger frequencies. We can specify this by $R \simeq R_K$ and by choosing the circuit parameters as follows:

$$|\kappa(\omega)|^2 = \begin{cases} |\kappa_0|^2 \approx\; 1\,, & \omega < \omega_0\,, \\ |\kappa_\infty|^2 \ll 1\,, & \omega > \omega_0\,. \end{cases} \tag{10}$$

This kind of frequency dependence is relevant in many experiments. High-temperature noise usually is avoided by applying various low-pass filtering techniques. The noise of the large impedances in these parts of the setup is cut at some low frequency ω_0 which is typical for the corresponding filter. Fluctuations from the rest of the circuit close to the tunneling device are unfiltered; they arise due to a low impedance $|\kappa_\infty|^2 R$ at the temperature $T_{\rm env}$. These fluctuations represent white noise since the junction capacitance cuts them only at the (very large) frequency $\hbar/(|\kappa_\infty|^2 RC) \gg k_{\rm B}T_{\rm env}$.

We can write $J(t)$ as a sum of a low-impedance part $\propto |\kappa_\infty|^2 R$ and the contribution of a high impedance $\propto |\kappa_0|^2 R \cdot \theta(\omega_0 - \omega)$. Then $P(E)$ can be calculated as the convolution $P(E) = \int {\rm d}\varepsilon\, P_{\rm low}(\varepsilon)\, P_{\rm high}(E-\varepsilon)$, where $P_{\rm low}(E)$ and $P_{\rm high}(E)$ are given by

$$P_{\rm low}(E) = \begin{cases} \dfrac{1}{\pi}\dfrac{\hbar\alpha_{\rm l}}{(\hbar\alpha_{\rm l})^2 + E^2}\,, & \hbar\alpha_{\rm l} > |E|\,, \\ \dfrac{2R|\kappa_\infty|^2}{R_K E}\,\dfrac{1}{1-\exp\left(-\beta_{\rm env}E\right)}\,, & \hbar\alpha_{\rm l} < |E|\,, \end{cases} \tag{11}$$

$$P_{\rm high}(E) = \frac{1}{2\hbar\alpha_{\rm h}\sqrt{\pi}}\exp\left[-\left(\frac{E-(2R/R_K)\hbar\omega_0}{2\hbar\alpha_{\rm h}}\right)^2\right]\,. \tag{12}$$

In order to obtain $P_{\rm high}(E)$ we have to assume that the charging energy is smaller than the temperature of the environment $E_{\rm C} < k_{\rm B}T_{\rm env}$. Moreover, we defined the parameters

$$\alpha_{\rm l} = (2\pi|\kappa_\infty|^2 R/R_K)/(\hbar\beta_{\rm env})\,, \quad \alpha_{\rm h} = \sqrt{(2R/R_K)\hbar\omega_0 k_{\rm B}T_{\rm env}}/\hbar\,, \tag{13}$$

which characterize the widths of the distribution functions [11]. We note that the special energy dependence of $P_{\rm low}(E)$, $P_{\rm high}(E)$ has its origin in the time dependence of $J(t) = J_{\rm low}(t) + J_{\rm high}(t)$, so that $\exp\left[J(t)\right] \propto \exp\left[-(\alpha_{\rm l}|t| + \alpha_{\rm h}t^2)\right]$. As we have discussed before, the term $\propto |t|$ corresponds to an essentially frequency-independent voltage-voltage correlator and characterizes white noise. Analogously we can attribute the term $\propto t^2$ in $J(t)$ to Gaussian-correlated noise.

In fig. 2 we show the numerically calculated tunneling rate in a normal junction for different ratios $\alpha_{\rm l}/\alpha_{\rm h}$. We have chosen $T_{\rm el} \ll E_{\rm C} < T_{\rm env}$ which is the experimentally relevant case. As long as $\alpha_{\rm l} \geq \alpha_{\rm h}$ the low impedance determines the behavior of the rate. For $V > 0$ the rate remains unchanged while for $V < 0$ it falls off $\propto \exp\left[-eV/k_{\rm B}T_{\rm env}\right]$, *i.e.* it decreases quite slowly due to the high temperature $T_{\rm env}$. This is important for high-sensitivity applications of single-electron devices [7],[8],[10]. With increasing $\alpha_{\rm h}$ the Coulomb blockade is found for large voltages. The energy provided by the fluctuations lifts the blockade for smaller voltages; for $V < 0$ the rate decreases even more slowly than in the low-impedance case.

Summarizing, we can say that we have presented a method to determine the influence on single-electron tunneling of an electromagnetic environment which is not in thermal equilibrium and, therefore, produces non-thermal noise. We stress, however, that the method *does not* apply to arbitrary noise such as, *e.g.*, shot noise.

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[11] For simplicity we have introduced the same temperature T_{env} in the high- and low-impedance part; the method, of course, is not restricted to this case.