Algebraic surfaces

SoSe 2021, University of Regensburg

Preliminary version

Introduction

The seminar is aimed to cover the methods and the results of the birational classification of algebraic surfaces (over an algebraically closed field of characteristic 0). This topic can be seen either as a first hardworking result in algebraic geometry that uses many techniques usually developed in the first courses in algebraic geometry, or as an introduction to the Mori program (and minimal model program) of birational classification of algebraic varieties of higher dimensions.

Plan of the seminar

Details of the talks

1 Numerical invariants of surfaces and first examples

Arithmetic and geometric genera, the irregularity, n-genus and Kodaira dimension of surfaces. Examples [ShaIsk, Ch. 1,2].

2 Curves on surfaces

Algebraic and rational equivalence of divisors on surfaces, Picard and Albanese varieties, the Neron-Severi group [ShaIsk, Ch. 3].

3 Intersection theory on surfaces

Intersection numbers of curves, the adjunction formula and the Riemann-Roch theorem.

Consider the material covered in [ShaIsk, Ch. 4, 5.1], take the proofs from [Har, V.1] whenever possible.

4 Numerical equivalence and the effective cone

Hodge index theorem, Nakai-Moisheson and Kleiman ampleness criteria [ShaIsk, Ch. 5].

What happens to the cone after a blow-up?

5 Birational maps between surfaces

Blow-up at a point, Castelnuovo-Enriques criterion of contractibility, *du Val singularities [ShaIsk, Ch. 6].

6 Del Pezzo surfaces

Cubic surfaces and the configuration of lines on them [ShaIsk, Ch. 13.3], [Reid, Ch. 1], del Pezzo surfaces of degrees at most 4 [Kol, III.3 up to Theorem 3.7].

7 Ruled surfaces

Surfaces that are projectivisations of rank 2 vector bundles on curves [Har, V.2] (cf. [Reid, Ch. 2]).

8 Minimal models and Mori fibre spaces

[ShaIsk, Ch. 7] (cf. [Reid, Ch. D])

9 Birational classification

[ShaIsk, Ch. 8], [Reid, Ch. E].

10 *Sarkisov program in dimension 2: Noether-Castelnuovo theorem

This topic may be left out if there are not enough speakers.

The classical Noether-Castelnuovo theorem describes the generators of the group of birational transformations of a rational surface. We will view it as a part of Sarkisov program that predicts that birational morphisms between Mori fibre spaces are generated by some elementary transformations called 'links'.

In order to do that we will have a glimpse at canonical singularities and Noether-Fano inequalities [And, 3.4] (cf. [Mat, Th. 1-8-8]).

11 Elliptic surfaces

A surface is elliptic if there is a surjective morphism to a smooth curve with connected fibres and the generic fibre of genus 1. In other words, this is 1-dimensional family of curves of genus 1 (not neccessarily smooth) [ShaIsk, Ch. 10].

12 Surfaces of canonical dimension 0

These are K3 surfaces, Enriques surfaces, abelian surfaces and bielliptic surfaces [ShaIsk, Ch. 11, 12].

13 Surfaces of general type

Surfaces of general type (or of Kodaira dimension 2) have moduli spaces that can be constructed using the canonical classes very much like in the case of curves. Apart from that we will discuss which numerical invariants can be realised for surfaces of general type and what kind of geometric restrictions they bring [ShaIsk, Ch. 9].

14 *Higher-dimensional perspectives

This topic may be left out if there are not enough speakers.

TBA

References

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[Har] Hartshorne, Robin. Algebraic geometry. Vol. 52. Springer Science & Business Media, 2013.

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[And] Andreatta, Marco. "An introduction to Mori theory: the case of surfaces." preprint, Universita di Trento, Trento (2003).