Übungen zu Quantenfeldtheorie

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Hint:
In all three exercises you should write the kinetic term plus the gauge fixing term in the action as

$$\int d^4x \int d^4y\ A^\mu(x) K_{\mu\nu}(x,y) A^\nu(y), \tag{1}$$

where

$$K_{\mu\nu}(x,y) = \int \frac{d^4p}{(2\pi)^4} D^{-1}_{\mu\nu}(p) e^{-ip(x-y)}. \tag{2}$$

Proceed to invert the matrix $D^{-1}_{\mu\nu}(p)$ to get the momentum space propagator $D_{\mu\nu}(p)$.
For this you should make an Ansatz for $D^{-1}_{\mu\nu}(p)$ in terms all the Lorentz structures that can appear, e.g. for Aufgabe 2

$$D_{\mu\nu}(p) = A g_{\mu\nu} + B p_\mu p_\nu + C n_\mu n_\nu + D p_\mu n_\nu + E n_\mu p_\nu, \tag{3}$$

where $A, ..., E$ are functions of the Lorentz scalars that have to be determined.

Aufgabe 1: The propagator for a massive vector field

For a massive abelian vector field V_μ, the free Lagrangian is given by

$$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} M^2 V_\mu V^\mu \tag{4}$$

Show that the propagator is given by

$$D_{\mu\nu}(k) = \frac{-i}{k^2 - M^2 + i\epsilon} \left[g_{\mu\nu} - \frac{k_\mu k_\nu}{M^2} \right]. \tag{5}$$

Aufgabe 2: gauge boson propagator in the axial gauge

Consider a massless abelian vector field

$$\mathcal{L}_{A,\ \text{kinetic}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \tag{6}$$

The axial gauge condition is given by

$$n^\mu A_\mu = 0 \tag{7}$$

where n_μ with $n^2 < 0$ is a space-like vector. Use the Faddeev-Popov method (insertion of a delta-function in the path-integral) to implement axial gauge. You should get

$$\mathcal{L}_{A,\ \text{kinetic + gauge fixing term}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2\xi} (n \cdot A)^2. \tag{8}$$

Calculate the propagator of a gauge boson in the axial gauge. You should get

$$D_{\mu\nu}(k) = \frac{-i}{k^2 + i\epsilon}\left[g_{\mu\nu} - \frac{1}{k \cdot n}(n_\mu k_\nu + k_\mu n_\nu) - \frac{\xi k^2 - n^2}{(k \cdot n)^2} k_\mu k_\nu\right]. \tag{9}$$

Aufgabe 3: gauge boson propagator in the Coulomb gauge

Consider again a massless abelian vector field, see (6). Use the Fadeev-Popov method to implement Coulomb gauge

$$\vec{\partial} \cdot \vec{A} = 0 \tag{10}$$

to get a Lagrangian analogous to (8) and calculate the propagator. To solve this problem, we suggest rewriting this gauge condition as

$$\partial_\mu A^\mu - (c_\mu \partial^\mu)(c_\nu A^\nu) = 0 \quad \text{where} \quad c_\mu = (1, 0, 0, 0). \tag{11}$$

Then make the same Ansatz as (3) with $n \to c$.
I get

$$D_{\mu\nu}(k) = \frac{-i}{k^2 + i\epsilon}\left[g_{\mu\nu} + \frac{(c \cdot p)^2 - p^2(1-\xi)}{(p^2 - (c \cdot p)^2)^2} p_\mu p_\nu + \frac{c \cdot p}{p^2 - (c \cdot p)^2}(c_\mu p_\nu + c_\nu p_\mu)\right]. \tag{12}$$