Übungen zu Quantenfeldtheorie

Prof. Dr. V. Braun
Jakob Schönleber

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Problem 1

Consider the Wilsonian effective action S_{eff} in eqs. (3.45) in the script. Recall that we are considering a Euclidean metric $g_{\mu\nu} = \delta_{\mu\nu}$ in d dimensions.
Calculate the leading contribution to δZ, the renormalization of the kinetic term

$$S_{eff} = \int d^d x \left[\frac{1}{2}(1+\delta Z)(\partial_\mu \Phi)^2 + \ldots\right] \tag{1}$$

Problem 2

Following the discussion in the lecture, section 3.4. Show that the propagator of the α-particle in momentum space is given by

$$D(p) = -\frac{2}{\Gamma(p)}, \tag{2}$$

where

$$\begin{aligned} \Gamma(p) &= \int \frac{d^2q}{(2\pi)^2} \frac{1}{[q^2+m^2][(p+q)^2+m^2]} \\ &= \frac{1}{2\pi} \frac{1}{\sqrt{p^2(p^2+4m^2)}} \ln \frac{\sqrt{p^2+4m^2}+\sqrt{p^2}}{\sqrt{p^2+4m^2}-\sqrt{p^2}}. \end{aligned} \tag{3}$$

Hints:
- Look for the quadratic in α_q term in $S_{eff}\left[\sqrt{N}m^2 + \alpha_q\right]$ which has the form

$$-\frac{1}{2}\int d^d x \int d^d y \, \alpha_q(y) K(x,y) \alpha_q(x) \tag{4}$$

Determine the kernel K.
- D is the defined as the Fourier transform of K^{-1}.