Übungen zu Quantenfeldtheorie

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Problem 1

Consider a real massive scalar field ϕ with ϕ^4 potential in d space-time dimensions

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 \tag{1}$$

In the lecture you have shown

$$(\partial_\mu \partial^\mu + m^2)\langle \Omega | T\{\phi(x)\phi(y)\}|\Omega\rangle = -\frac{\lambda}{3!}\langle \Omega | T\{\phi^3(x)\phi(y)\}|\Omega\rangle - i\delta^4(x-y) \tag{2}$$

using the path integral representation.

a) Verify eq. (2) diagrammatically to first order in λ.

b) Give a diagrammatical argument for eq. (2) that applies to all orders in λ.

Problem 2

Consider the theory with the Lagrangian density

$$\mathcal{L}' = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{3}{2\lambda}\sigma^2 - \frac{1}{2}\sigma\phi^2, \tag{3}$$

where σ is a static scalar field (i.e. has no kinetic term).

a) Show that classically the Lagrangian $\mathcal{L}'$ in eq. (3) is equivalent to $\mathcal{L}$ in eq. (1).

b) Using the path integral representation, show that $\mathcal{L}'$ is equivalent to $\mathcal{L}$, also in the quantum theory. That is, you need to show that the n-point correlation functions

$$\langle \Omega | T\{\phi(x_1)...\phi(x_n)\}|\Omega\rangle \tag{4}$$

are the same.

Hint: Make a suitable change of variables $\sigma \to \sigma + ?$ in the path integral.

Remark: Removing a particle from the theory in this way is commonly referred to as "integrating out" that particle. In this case it was easy, since σ was a static field.

Problem 3

Consider again the Lagrangian $\mathcal{L}'$ in eq. (3).

a) Show that the free propagator of the σ particle reads

$$\langle 0 | T\{\sigma(x)\sigma(y)\}|0\rangle = \int \frac{d^d p}{(2\pi)^d i}\, e^{-ip(x-y)}\, (-\frac{1}{3}\lambda) = \frac{i\lambda}{3}\delta^{(d)}(x-y). \tag{5}$$

Hint: You may add a term $\frac{\epsilon}{2}(\partial_\mu \sigma)^2$ to $\mathcal{L}'$ and then let $\epsilon \to 0$ later.

b) Give a diagrammatical argument that the quantum theories of $\mathcal{L}'$ and $\mathcal{L}$ are equivalent.