Übungen zu Quantenfeldtheorie

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e^+e^- scattering in the Standard Model

The photon ($Z-$boson) interaction with fermions is given by the following Lagrangian

$$L_{\bar{f}f\gamma} = ieQ_f \bar{f}\gamma_\mu A^\mu f\,, \tag{1}$$

$$L_{\bar{f}fZ} = \frac{ie}{4s_w c_w}\bar{f}\gamma_\mu(g_V^f + g_A^f\gamma_5)Z^\mu f\,. \tag{2}$$

Here Q_f is the charge of a fermion f in the unit of the electron charge ($Q_{e^-} = 1$).
$c_w = \cos\theta_W$ and $s_W = \sin\theta_W$, where θ_W is the Weinberg angle.
The vector and axial-vector couplings satisfy the following relation

$$\frac{g_V^f}{g_A^f} = 1 - 4|Q_f|\sin\theta_W\,. \tag{3}$$

For example,

$$g_V^e = -1 + 4\sin^2\theta_W\,, \qquad g_A^e = -1\,. \tag{4}$$

- Show that at the leading order the unpolarized cross-section for

  $$e^+e^- \xrightarrow{\gamma,Z} f\bar{f}$$

  scattering at $s \simeq M_Z^2$, where M_Z is the mass of the Z-boson, is given by the following expression (neglect fermion masses and all terms which are of order $\mathcal{O}(s - M_Z^2)$)

  $$\frac{d\sigma(s)}{d\cos\theta} = \sigma(s)\left[\frac{3}{8}(1+\cos^2\theta) + A_{FB}^f\cos\theta\right]\,, \tag{5}$$

  where

  $$A_{FB}^f = \frac{3}{4}A_e A_f, \qquad (FB = \text{forward-back asymmetry}) \tag{6}$$

  $$A_f = 2\frac{g_V^f g_A^f}{(g_V^f)^2 + (g_A^f)^2}\,. \tag{7}$$

- Calculate the cross section in the limit $s \gg M_Z^2$ (i.e. derive an analog of Eq. (5)).