Übungen zu Quantenfeldtheorie

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Axial anomaly from triangle diagrams

For this exercise, consider Quantum Electrodynamcs. Let

$$V_\mu(x) = \bar{\psi}(x)\gamma_\mu\psi(x), \qquad \text{vector current}$$
$$A_\mu(x) = \bar{\psi}(x)\gamma_\mu\gamma_5\psi(x), \qquad \text{axial current}$$
$$P(x) = \bar{\psi}(x)i\gamma_5\psi(x) \qquad \text{pseudoscalar current} \tag{1}$$

Classically, the vector current is conserved and the axial current is conserved if electron mass is set to zero. We have shown in the Lectures that in quantum field theory

$$\partial^\mu V_\mu = 0\,,$$
$$\partial^\mu A_\mu = \frac{g^2}{8\pi^2}F_{\alpha\beta}\widetilde{F}^{\alpha\beta} \qquad \text{Adler-Bardeen Anomaly} \tag{2}$$

where the first statement is valid for arbitrary electron mass and the second one assumes $m = 0$. The aim of this exercise is to show how these results arise in perturbation theory on diagrammatic level.

For this purpose, consider (lowest order) contributions to the following three-point correlation functions:

$$\mathbb{V}_{\mu\nu}(k_1,k_2) = i\int d^4x_1 d^4x_2 \langle 0|T\{V_\mu(x_1)V_\nu(x_2)\partial^\lambda V_\lambda(0)\}|0\rangle e^{ik_1x_1+ik_2x_2}$$
$$\mathbb{A}_{\mu\nu}(k_1,k_2) = i\int d^4x_1 d^4x_2 \langle 0|T\{V_\mu(x_1)V_\nu(x_2)\partial^\lambda A_\lambda(0)\}|0\rangle e^{ik_1x_1+ik_2x_2} \tag{3}$$

- Draw the relevant Feynman diagrams are write down the corresponding expressions.
- Without doing an explicit calculation, show that naively, i.e. if all integrals were finite and well defined, $\mathbb{V}_{\mu\nu}(k_1,k_2) = 0$ and $\mathbb{A}_{\mu\nu}(k_1,k_2) = 0$ for $m = 0$.
- To calculate the diagrams more carefully one needs to introduce some regularization. Since extending the γ_5 matrix to $d = 4 - 2\epsilon$ dimensions involves some subtleties, do not use dimensional regularization for this calculation, but make use of the so-called Pauli-Villars regularization which is defined as follows:

  The idea is to suppress contributions of large momenta modifying the expression for the fermion propagators in loops as

$$\frac{1}{\not{p}-m} \mapsto \frac{1}{\not{p}-m} - \frac{1}{\not{p}-M} \tag{4}$$

that is, subtracting a contribution of a fermion with a very large mass M that serves as a regulator. It is obvious that with this modification the propagator vanishes for $p \gg M$ so that the high-momentum tails are subtracted. The renormalized amplitude, e.g. $\mathbb{A}_{\mu\nu}$ for the present case can be defined as the difference of amplitude calculated with the physical mass m and the regulator mass M,

$$\mathbb{A}_{\mu\nu}^{\text{reg}} = \mathbb{A}_{\mu\nu}(m) - \mathbb{A}_{\mu\nu}(M) \tag{5}$$

where one has to take the limit $M \to \infty$.

- Using this procedure, show that the result for the vector current does not change, but for axial current one obtains in the limit $M^2 \to \infty$ and $m^2 \to 0$

$$\mathbb{A}_{\mu\nu}^{\text{reg}}(k_1, k_2) = \frac{1}{2\pi^2}\epsilon_{\mu\nu\rho\sigma}k_1^\rho k_2^\sigma \tag{6}$$

- Show that this result is in agreement with the Adler-Bardeen anomaly (2).