Monads and their applications II 8

Exercise 1.
Let $\mathscr{A}$, $\mathscr{K}$ be 2-categories, $W\colon \mathscr{A}^{\mathrm{op}} \to \mathbf{Cat}$ and D, $D'\colon \mathscr{A} \to \mathscr{K}$ 2-functors. Let $\alpha\colon \mathscr{D} \to \mathscr{D}'$ be an equivalence. Show that if W is a flexible weight and $\alpha_a\colon D_a \to D'_a$ is an equivalence for each $a \in \mathscr{A}$, then the induced 1-cell

$$W \odot_{\mathscr{A}} \alpha\colon W \odot_{\mathscr{A}} D \to W \odot_{\mathscr{A}} D'$$

is an equivalence.

Exercise 2.
Each bilimit $\{W, D\}_b$ comes with a unit $\zeta\colon W \to \mathscr{K}(\{W, D\}_b, D-)$ (corresponding to the identity of $\{W, D\}_b$).

If $(\{W, D\}'_b, \zeta')$ is another bilimit, let $\mathscr{C}$ be the category whose objects are pairs (f, φ) of an equivalence $f\colon \{W, D\}_b \to \{W, D\}'_b$ and an invertible modification φ between the resulting triangle involving ζ, ζ', and f. The morphisms are modifictions between the equivalences satisfying the evident equation. Show that $\mathscr{C}$ is a contractible groupoid, that is, $\mathscr{C}$ is non-empty and there is a unique isomorphism between any two equivalences.

Exercise 3.
A weight $W\colon \mathscr{A}^{\mathrm{op}} \to \mathbf{Cat}$ is called *sifted* if $W \odot_{\mathscr{A}} -\colon [\mathscr{A}, \mathbf{Cat}] \to \mathbf{Cat}$ preserves finite products. Show that W is sifted if and only if it preserves the terminal object and binary products of representable presheaves.