Monads and their applications II 5

Exercise 1.
Show that the category of codescent data on a truncated simplicial diagram $X_\bullet \colon \Delta_{\leq 2}^{\mathrm{op}} \to \mathscr{K}$ on the object C is naturally isomorphic to the 2-functor

$$[\Delta_{\leq 2}, \mathbf{Cat}]\big(W, \mathscr{K}(X_{(-)}, C)\big),$$

where $W \colon \Delta_{\leq 2} \to \mathbf{Cat}$ denotes the inclusion. Thus the codescent object of $X_\bullet$ is precisely the W-weighted colimit of $X_\bullet$. Find an analogous weight for iso-codescent objects.

Exercise 2.
Show that a category with coinserters and coequifiers has codescent objects and iso-codescent objects.

Exercise 3.
Let X be a scheme and let $U_i \to X$ be a finite Zariski cover with U_i affine. Let $X_\bullet$ be the Čech nerve of this cover, so $X_0 = \coprod U_i$, X_1 is the coproduct of the intersections $U_i \cap U_j$, and so on. This defines a simplicial diagram of categories by sending X_i to $\mathbf{QCoh}(X_i)$. Since $\coprod U_i \to X$ defines an augmentation of this simplicial diagram, we have an induced functor from $\mathbf{QCoh}(X)$ to the category of descent data of $\mathbf{QCoh}(X_\bullet)$.

Let F be a quasi-coherent sheaf of $\mathcal{O}_X$-modules whose restriction to each U_i is finitely generated free of rank d. What does the descent datum of F induced by the above functor look like? (If you are unfamiliar with schemes, you can do this exercise in the affine case instead.)