# Monads and their applications II 3

**Exercise 1.**
Complete the proof that pseudomonoids in a locally presentable (strict) monoidal 2-category are 2-monadic. This entails checking the 2-functoriality of $\kappa_1$ and $\kappa_2$ and completing the construction of the $\beta_i$ (the "unit triangle" part).

**Exercise 2.**
Let $\mathscr{K}$ be a symmetric monoidal locally $\kappa$-presentable 2-category such that $X \otimes -$ is $\kappa$-accessible for all $X \in \mathscr{K}$. Show that the 2-categories of braided pseudomonoids and symmetric pseudomonoids (with braided respectively symmetric lax monoidal morphisms and 2-cells) are isomorphic to categories of the form $T\text{-}\mathbf{Alg}_\ell$ for certain $\kappa$-accessible 2-monads $T$. In fact, show that these can be built using only co-iso-inserters and coequifiers from the 2-monad for pseudomonoids constructed in class.

**Exercise 3.**
Let $R$ be a commutative ring. Show that there is a finitary 2-monad on $\mathbf{Cat}_R$, the 2-category of small $R$-linear categories, whose algebras are symmetric monoidal $R$-linear categories $\mathscr{A}$ with (chosen) finite colimits with the additional property that $X \otimes -$ preserves finite colimits (*not* strictly, only up to isomorphism); the 1-cells are the $R$-linear symmetric monoidal functors which preserve finite colimits (again up to isomorphism), and the 2-cells are the symmetric monoidal natural transformations. (Hint: combine the 2-monad from Exercise 2 with the 2-monad for finite colimits constructed in class (forming a coproduct in the first step), and then use an inverter).

**Exercise 4.**
Let $T$ be a 2-monad on the 2-category $\mathscr{K}$ and let $A \in \mathscr{K}$ be an object. A *lax* $T$-algebra structure on $\mathscr{A}$ is a lax monad morphism $T \rightsquigarrow \langle A, A \rangle$. Use the definition of $\langle A, A \rangle$ as right 2-adjoint to $T \mapsto TA$ to unravel this definition in terms of a 1-cell $TA \to A$, various 2-cells in place of the strict algebra axioms, and coherence laws between these.

**Exercise 5.**
Show that a 2-category $\mathscr{K}$ with products, inserters, and equifiers, also has iso-inserters, comma-objects, and iso-comma-objects. Moreover, show that the power $\{C, X\}$ exists for any $X \in \mathscr{K}$ and any small category $C$. For the latter, consider the truncated nerve of $C$ consisting of the set of objects $C_0$, the set of arrows $C_1$, and the set of composable arrows $C_1 \times_{C_0} C_1$.