Objective Bayesian Epistemology for Inductive Logic on Predicate Languages

Jürgen Landes and Jon Williamson
Philosophy Department and Centre for Reasoning
University of Kent

May 7, 2013

Keywords: Objective Bayesianism, Inductive Logic, Scoring Rules

Main Objective. The main aim of this work is to provide a new justification of the three norms of objective Bayesian epistemology: that degrees of belief should be (i) probabilities, (ii) calibrated to evidence of physical probabilities, and (iii) sufficiently equivocal or non-extreme. While these norms are typically each justified in different ways, it is shown that they can be given a unified justification in terms of minimising worst-case expected loss.

Introduction. Objective Bayesianism, as developed in [10, 11], is based on three norms that a rational agent ought to adhere to when forming beliefs. These norms require that

1. beliefs should be probabilities (Probability),
2. beliefs should be calibrated to the agent's evidence of physical probabilities (Calibration),
3. otherwise beliefs should equivocate between the sentences of $\mathcal{L}$ (Equivocation).

The probability norm is usually justified by a Dutch Book argument which assumes that a rational agent avoids *sure* loss. The calibration norm, also known as the Principal Principle, is normally justified in a slightly different betting scenario with repeated bets. Here, the agent's rationality is capture by *long-run* or *expected* loss avoidance. Finally, the equivocation norm, which amounts to Shannon entropy maximization among the calibrated probability functions, can also be justified by considering a betting scenario. In this third scenario an agent aims to avoid *worst case* expected default loss.

In a forthcoming paper [4] we show how a single justification for all three norms can be given in terms of worst case expected default loss avoidance. This loss is computed employing scoring rules, which have become a popular method to assess probabilistic forecasters. [4] presents the argument in the context of beliefs defined over propositions or sentences of a propositional language. In contrast, in this paper we extend the argument to the greater expressive power of a first-order predicate language.

Related Work. Classically, the probability norm has normally been justified by Dutch Book Arguments or Cox's Theorem. As the arguments against Dutch Books mounted [1] a new approach has emerged. Based on Brier's original suggestion and Savage's seminal work [7] scoring rules have become a popular mean to justify the probability norm in belief formation and the calibration norm [2, 3, 5, 6, 9]. Scoring rules have found further applications, for instance in the assessment of probabilistic forecasters [8].

Predicate Languages. The probability norm and the calibration norm for *predicate* languages with countably many constants are well known and well understood. The equivocation norm for predicate languages does not directly generalize to predicate languages. Stating that extreme beliefs ought to be avoided where possible is straightforward; however a formal version of this statement requires some thought. Simply attempting to maximize the entropy

$$\sum_{\omega\in\Omega} -P(\omega)\log P(\omega)$$

fails at the first hurdle. On a predicate language any atomic statement ω has to decide infinitely many literals and is, because of its infinite length, *not* a well formed formula. Thus, $P(\omega)$ is not defined.

Fortunately, defining the equivocator $P_=$ (the probability function which is least committal) is uncontentious. $P_=$ is defined as the unique function which assigns every literal probability $\frac{1}{2}$ and which treats different literals as independent, and hence assigns a conjunction of n different literals probability $\frac{1}{2^n}$.

We may thus formalize the equivocation norm on predicates languages as saying that one should adopt a probability function as belief function which is as close as possible to $P_=$ while satisfying the constraints imposed by the evidence—see [10, Chapter 5.1] for details.

Armed with our formal results for propositional languages we shall give a single justification for the norms of Objective Bayesianism for predicate languages.

Consequences for Inductive Logic. An agent adopting the objective Bayesian approach to belief formation can answer the question of how strongly a consistent set of uncertain premisses support a conclusion. Firstly, premisses are interpreted as constraints on physical probabilities of the form $P^*(\varphi_i) \in X_i$, where P^* denotes physical probability, the φ_i are sentences over an appropriate language and the X_i are subsets of $[0,1] \subset \mathbb{R}$.

Next, the probability function $P^\dagger$ consistent with the evidence which maximizes entropy is computed, which is unique under mild assumptions. A conclusion ψ is then supported to a degree $P^\dagger(\psi) \in [0,1] \subset \mathbb{R}$.

Thus, objective Bayesianism can provide semantics for inductive entailment relationships of the form:

$$\varphi_1^{X_1}, \ldots, \varphi_n^{X_n} \mathrel{|\!\!\approx} \psi^Y \quad .$$

Here $Y = P^\dagger(\psi)$.

Future Work. In the near future we shall develop and implement a credal net algorithm that allows an efficient computation of the probability function $P^\dagger$ which maximizes entropy.

References

[1] Hájek, A. (2008). Arguments for - or against - Probabilism? *The British Journal for the Philosophy of Science*, 59(4):793–819.

[2] Joyce, J. M. (1998). A Nonpragmatic Vindication of Probabilism. *Philosophy of Science*, 65(4):575–603.

[3] Joyce, J. M. (2009). Accuracy and Coherence: Prospects for an Alethic Epistemology of Partial Belief. In Huber, F. and Schmidt-Petri, C., editors, *Degrees of Belief*, volume 342 of *Synthese Library*, pages 263–297. Springer.

[4] Landes, J. and Williamson, J. (2013). Objective Bayesianism and the maximum entropy principle. *Entropy*, 15.

[5] Leitgeb, H. and Pettigrew, R. (2010a). An Objective Justification of Bayesianism I: Measuring Inaccuracy. *Philosophy of Science*, 77(2):201–235.

[6] Leitgeb, H. and Pettigrew, R. (2010b). An Objective Justification of Bayesianism II: The Consequences of Minimizing Inaccuracy. *Philosophy of Science*, 77(2):236–272.

[7] Savage, L. J. (1971). Elicitation of personal probabilities and expectations. *Journal of the American Statistical Association*, 66(336):783–801.

[8] Schervish, M. J., Seidenfeld, T., and Kadane, J. B. (2009). Proper Scoring Rules, Dominated Forecasts, and Coherence. *Decision Analysis*, 6(4):202–221.

[9] Seidenfeld, T. (1985). Calibration, Coherence, and Scoring Rules. *Philosophy of Science*, 52(2):274–294.

[10] Williamson, J. (2010). *In Defence of Objective Bayesianism.* Oxford University Press.

[11] Williamson, J. (2013). From Bayesian epistemology to inductive logic. *Journal of Applied Logic*, DOI 10.1016/j.jal.2013.03.006.