

ON THE LOGICAL STRUCTURE OF DE FINETTI'S NOTION OF EVENT

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This paper aims to (i) provide a logical development of Bruno de Finetti's conception of events and (ii) suggest that the subjective nature of de Finetti's interpretation of probability emerges in a clearer form against such a logical background. This provides grounds for more general considerations on the subtle relation between (subjective) probability and logic.

We begin by recalling the centrality of the notion of event, which constitutes one of the main themes of de Finetti's own philosophical reflections on the theory of probability (de Finetti, 2008). For him an event is any random variable which takes values on the binary set and which, in addition, satisfies the properties of being a *single* and *well-defined* case. Single is opposed to “repeatable”, and this marks the clear and well-known separation between the subjectivist and the frequentist interpretations of probability. The second requirement has attracted less attention despite the crucial role it plays in making probability subjective. Consider two idealised agents called Bookmaker and Gambler. For de Finetti an event is “well-defined” when it stands for a question for which (a) neither Gambler nor Bookmaker have a definite answer and (b) Gambler and Bookmaker agree on the conditions under which this question will be answered:

[T]he characteristic feature of what I refer to as an “event” is that the circumstances under which the event will turn out to be “verified” or “disproved” have been fixed in advance.
(de Finetti, 2008, p. 150)

We work in a propositional setting, with a finite language \mathcal{L} and we denote by $\mathcal{S}\mathcal{L}$ the set of sentences built up, as usual, from \mathcal{L} . In order to address the question as to whether a sentence $\theta \in \mathcal{S}\mathcal{L}$ is “well-defined” in the above sense –and hence it can be said to represent an event– we suppose that Gambler finds herself in a certain epistemic state $w \in W$ which is defined as a *partial* valuation on \mathcal{L} . We assume that epistemic states are dynamic in the sense that some propositional variables which are not decided at w may be decided at future states, until all propositional variables in \mathcal{L} are

eventually decided. Yet not all epistemic states in W may be accessible (in the usual modal sense) by Gambler at w . This allows for the possibility that Gambler may never be in a position to ascertain whether θ is decided positively or negatively.

This background allows us to put forward a definition of *events* which is relative to the *state of information* of the individuals involved in the betting problem. Let $\theta \in \mathcal{SL}$. We say that θ is

- a *w-fact* if the truth value of θ is determined at w ;
- a *w-event* if it is not a *w-fact*, but there exists a w -accessible $w' \in W$ which determines the truth value of θ ;
- *w-inaccessible* if no state w' which decides θ is accessible from w .

A suitable formalisation of the above allows us to introduce *Bet functions*

$$Bet : \mathcal{SL} \rightarrow [0, 1]$$

which are defined by letting

$$Bet(\theta) = \begin{cases} \epsilon \in \{0, 1\} & \text{if } \theta \text{ is a fact} \\ 0 & \text{if } \theta \text{ is inaccessible} \\ x \in [0, 1] & \text{if } \theta \text{ is an event} \end{cases} .$$

Our central result is a refinement of the classical representation theorem for probability functions (see, e.g. [Paris, 1994](#)). In particular we show that whilst all coherent bet functions are probabilities, *probabilities which are defined on sentences which are not events can coherently only be given trivial values*. Trivial, in this context, means one of two things. Either a sentence can (coherently) be given only its truth value (and this characterises betting on facts), or it should be given 0. This means that the “uncertainty mass” must be concentrated only on events. Since events are defined relative to the agents’ own epistemic state, this determines to a crucial extent the subjective nature their rational degrees of belief.

REFERENCES

- B. de Finetti. *Philosophical lectures on probability*. Springer Verlag, 2008.
- J.B. Paris. *The uncertain reasoner’s companion: A mathematical perspective*. Cambridge University Press, 1994.

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