# Uncertain deductive reasoning

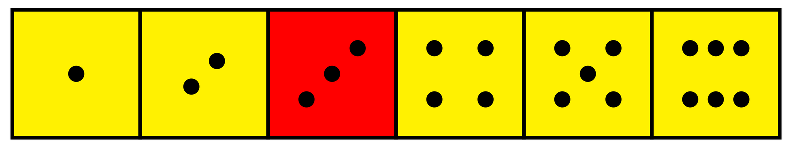

Niki Pfeifer & Gernot D. Kleiter
Department of Psychology, University of Salzburg

www.users.sbg.ac.at/~pfeifern/


## Introduction

Consider a fair dice. *What's the following probability?*

**P( If** it's a 3, **then** it's an even number.**)** = ...



| | | | | | | | |
|---|---|---|---|---|---|---|---|
| it's a 3 | 0 | 0 | 1 | 0 | 0 | 0 | |
| it's even | 0 | 1 | 0 | 1 | 0 | 1 | |
| Mat. conditional: 3 ⊃ even | 1 | 1 | 0 | 1 | 1 | 1 | 5/6 |
| Conditional event: even \| 3 | i | i | 0 | i | i | i | 0 |

- Using a coherence based probability logic framework to model human inference ("Mental probability logic", [1-4])
- Investigating empirically probabilistic argument forms
- Deductive consequence relation:

$\mathfrak{P}_1$ $P($**If** shape X is a triangle, **then** shape X is blue.$) = x$
$\mathfrak{P}_2$ $P($Shape X is a triangle.$) = y$
——————————————— log. valid
$\mathfrak{C}$ $P($Shape X is blue.$) \in [xy\,,\, xy+1-y]$

. . . the uncertainty in the premises is transmitted deductively to the uncertainty of the conclusion (not to be confused with probabilistic consequence relations, like $P(\mathfrak{C}|\mathfrak{P}_2)$).

- **Goal:** Building a competence theory of human reasoning

## Experiment I: Two paradoxes of ⊃

| | | |
|---|---|---|
| Paradox 1: | $B \quad \therefore A \supset B$ | (logically valid) |
| | $P(B) = x \therefore P(A \supset B) \in [x, 1]$ | (prob. informative) |
| | $P(B) = x \therefore P(A \wedge B) \in [0, x]$ | (prob. informative) |
| | $P(B) = x \therefore P(B\|A) \in [0, 1]$ | (prob. non-informative) |
| Paradox 2: | $\neg A \quad \therefore A \supset B$ | (logically valid) |

**Example item:** $B \therefore$ **If** $A$, **then** $B$ **(Paradox 1, P$_{90}$)**

[A] Simon is 90% certain: There is a square on this card.

*Considering [A], how certain can Simon be that the following sentence is true?*

**If** there is a red shape on this card, **then** there is a square on this card.

Considering [A], can Simon infer—at all—how certain he can be, that the sentence in the box is true?

☐ NO, Simon cannot infer his certainty, since everything between 0% and 100% is possible.

☐ YES, Simon can infer his certainty. *In case you ticked YES, please fill in:*

Simon can be certain from at least ___% to at most ___%, that the sentence in the box is true. 0 ——— 100 %

### Results ("Paradox 1": $n_1 = 16$, "Paradox 2": $n_2 = 15$)

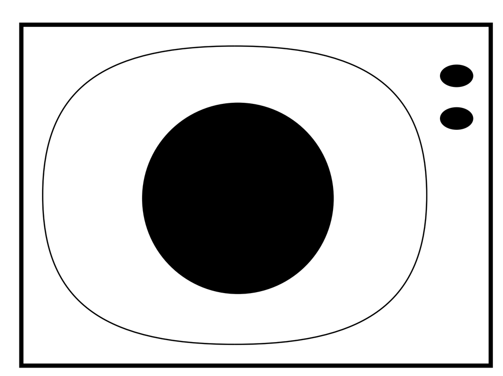
| | P$_{60}$ | P$_{70}$ | P$_{90}$ | P$_{vl}$ | P$_{ac}$ | MP$_{90}$ | MP$_{70}$ | MP$_{80}$ | MP$_{vl}$ | MP$_{ac}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| % correct per task (conclusion: **If** $A$, **then** $B$) | | | | | | | | | | |
| Paradox 1 | 62.50 | 81.25 | 68.75 | 68.75 | 68.75 | 62.50 | 87.50 | 81.25 | 75.00 | 93.75 |
| Paradox 2 | 73.33 | 73.33 | 73.33 | 80.00 | 66.67 | 73.33 | 73.33 | 86.67 | 80.00 | 93.33 |
| % correct per task (conclusion: **If** $A$, **then** not $B$) | | | | | | | | | | |
| Paradox 1 | 75.00 | 68.75 | 62.50 | 75.00 | 43.75 | 81.25 | 87.50 | 87.50 | 68.75 | 87.50 |
| Paradox 2 | 86.67 | 86.67 | 86.67 | 66.67 | 66.67 | 80.00 | 86.67 | 73.33 | 93.33 | 93.33 |

- most participants understand that the paradoxes are probabilistically **non**-informative
- evidence for the conditional probability interpretation of the conditional; no evidence for implicit and fully explicit mental models

## Experiment II: Representation of "if–then"

**Example item: Subject/Predicate condition, AA**

**If** there is a circle on the screen, **then** the circle is black.



Does the shape on the screen speak for the assertion in the box?

☐ speaks against ☐ neither/nor ☐ speaks for

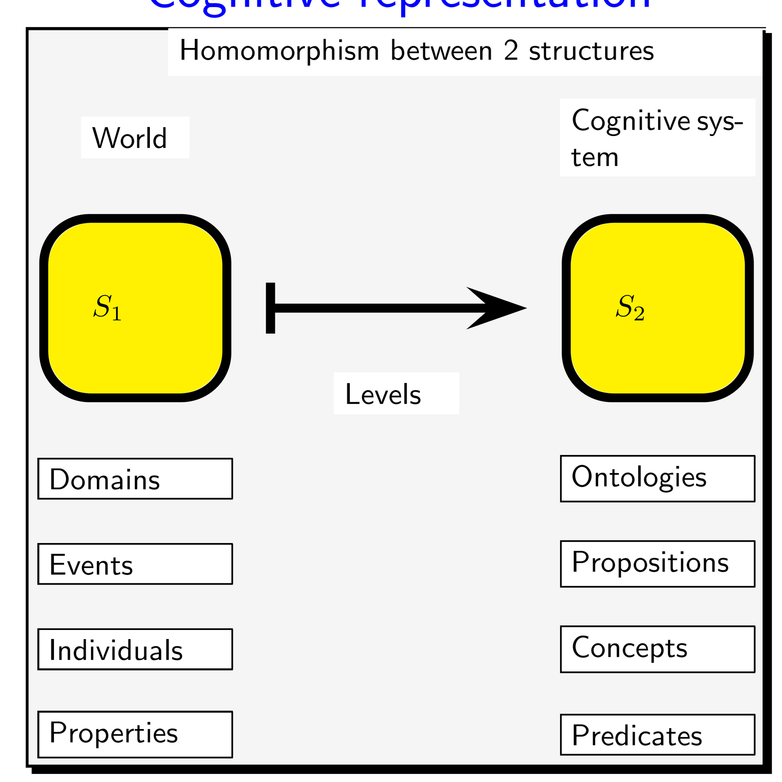
### Results: Mean response percentages

| Condition | Response | AA | AN | NA | NN |
|---|---|---|---|---|---|
| Sub./Pred. ($n_1 = 18$) | speaks against | 2.78 | 86.11$^{\| \wedge \supset}$ | 30.56$^{\wedge}$ | 22.22$^{\wedge}$ |
| | neither/nor | 4.17 | 11.11 | **61.11**$^{\|}$ | **76.39**$^{\|}$ |
| | speaks for | 93.06$^{\| \wedge \supset}$ | 2.78 | 8.33$^{\supset}$ | 1.39$^{\supset}$ |
| Pred./Subj. ($n_2 = 18$) | speaks against | 0.00 | 91.67$^{\| \wedge \supset}$ | **58.33**$^{\wedge}$ | 47.22$^{\wedge}$ |
| | neither/nor | 5.56 | 6.94 | 26.39$^{\|}$ | 50.00$^{\|}$ |
| | speaks for | 94.44$^{\| \wedge \supset}$ | 1.39 | 15.28$^{\supset}$ | 2.78$^{\supset}$ |

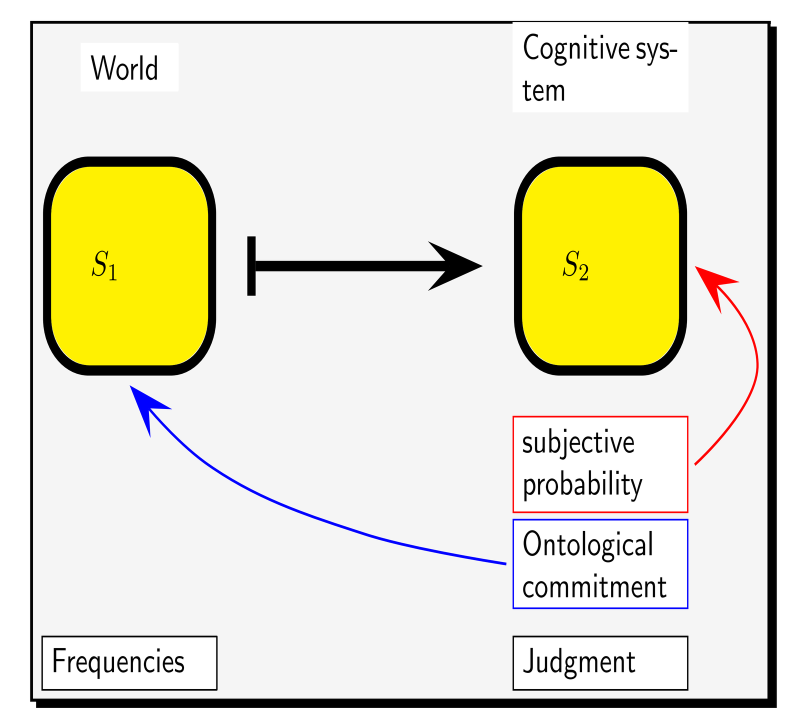
- Most participants in the Subject/Predicate condition represent the conditional as a conditional event, $(\cdot|\cdot)$
- Why is there an asymmetry between the Subject/Predicate condition and the Predicate/Subject condition?

## Cognitive representation of subjective probabilities

Cognitive representation




Subjective probability

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**References**

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