

Mental Probability Logic—A Competence Model of Human Rationality

NIKI PFEIFER

Department of Psychology, University of Salzburg
Hellbrunnerstrasse 34, 5020 Salzburg, Austria

<niki.pfeifer@sbg.ac.at>

1. Introduction

Question

What is an appropriate normative standard of reference for evaluating the rationality of human reasoning?

Ideas

- *Problem*: Classical logic is too strict and monotone.
- Traditional theories of human reasoning use wrong rationality norms!
- *Nonmonotonicity*: People withdraw conclusions.
- Soft version of *if-then*, that allow for *exceptions*:

$$\text{If } A, \text{ normally } B \quad \Leftrightarrow \quad P(B|A) > .5$$

2. Human Nonmonotonic Reasoning

Introduction

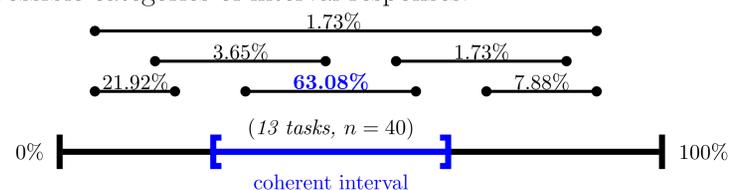
- SYSTEM P is a set of basic rationality postulates every system of nonmonotonic reasoning should satisfy (Kraus, Lehmann & Magidor, 1990).
- SYSTEM P combines logic and probability by inference rules, like $P(B|A)=x$, $P(C|A)=y \Rightarrow P(B\&C|A) \in [\max(0, x+y-1), \min(x, y)]$.

Method (Pfeifer & Kleiter, 2005a, 2006)

You know about a tourist party from Alsace:
70% of the tourist party speak *French*.
60% of the tourist party speak *German*.
How many of the tourist party speak *both French and German*?

Results

Six possible categories of interval responses:



References

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- Pfeifer, N., & Kleiter, G. D. (2005c). Towards a mental probability logic. *Psychologica Belgica*, 45(1), 71-99.
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- **Nonmonotonic** inference rules are valid in SYSTEM P and are **endorsed** by the subjects: the majority infers *coherent* intervals.
- **Monotonic** inference rules are not valid in SYSTEM P and are **not endorsed**: the subjects infer wide and not informative intervals.

3. Mental Probability Logic (Pfeifer & Kleiter, 2005c, 2005b)

Mental probability logic is a competence theory about how humans interpret and reason about common sense conditionals. Common sense conditionals are interpreted as “high” coherent conditional probabilities.

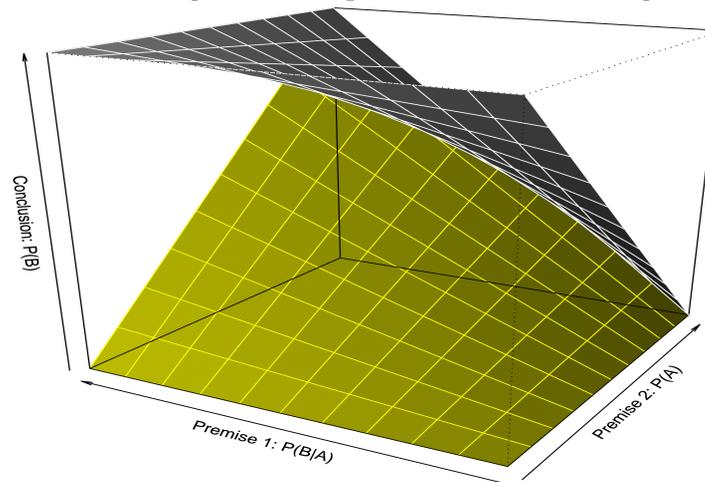


Figure: coherent intervals of the probabilistic MODUS PONENS. Coherent lower probability bounds are in green. Coherent upper probability bounds are in grey.

- Conditional probability logic is the normative standard of reference.
- Conditional probability logic interprets inference rules of logic probabilistically, e.g., the MODUS PONENS:

	Logic	Conditional probability logic
Premise 1:	$A \rightarrow B$	$P(B A) = x$
Premise 2:	A	$P(A) = y$
Conclusion:	B	$P(B) \in [xy, 1 - y + xy]$
	(log. valid)	(prob. informative)

- There are logically valid inference rules that are probabilistically not informative: e.g., CONTRAPOSITION, TRANSITIVITY.
- There are logically not valid inference rules that are probabilistically informative: e.g., DENYING THE ANTECEDENT.
- If subjects infer coherent intervals in the DENYING THE ANTECEDENT, they are *not* irrational—as usually claimed in classical

logic based theories of reasoning.

4. Syllogistic Reasoning (Pfeifer, 2006)

Originated by Aristotle (*384, †322), syllogisms are special argument forms with two premises and one conclusion. Since Störing (1908), syllogisms are prototypical reasoning tasks in psychology.

Problem

The universal quantifier is too strong, while the existential quantifier is too weak. Thus, classical syllogistics is not an appropriate rationality norm.

Solution

Replace the classical quantifiers by “realistic” quantifiers (“Most”, “Almost all”, “All, except x ”, “ x/n ”), e.g.,

$$\begin{array}{l} \text{Most birds have wings.} \quad |\{B \cap W\}| > |\{B \cap \bar{W}\}| \\ \text{Most birds have feathers.} \quad |\{B \cap F\}| > |\{B \cap \bar{F}\}| \\ \text{At least one thing has feathers and wings.} \quad |\{F \cap W\}| > 0 \end{array}$$

... reasoning about **frequencies**.

5. Concluding Remarks

- Behavior can only be said to be **rational** with respect to a given normative standard of reference.
- The traditional normative standards of reference are **too strong**.
- Conditional probability logic is the proposed normative standard.
- **Mental probability logic**
 - is proposed as a competence theory of human reasoning
 - combines ideas from nonmonotonic reasoning, logic, and probability theory (coherence, probability intervals)
 - interprets “*if A, then B*” as $P(B|A)$
 - is about probability propagation from the premises to the conclusion