Building bridges between the world and claims about it:
Commentary on Lumer’s Paper “Strength of justification. The rational degree of certainty approach”

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“The sentences of logic are tautologies. Therefore, the sentences of logic say nothing.” (Ludwig Wittgenstein, 1922, *Tractatus logico-philosophicus*)

1. LOGIC ALONE IS NOT ENOUGH FOR THE EPISTEMOLOGY OF ARGUMENTATION
From an argumentation theoretical point of view, this quote from Wittgenstein’s *Tractatus* can be interpreted to mean that classical first order logic (short: “logic”) alone is not enough for the rational reconstruction of common sense epistemic arguments. Why? Rational epistemic argumentation is about claims which can in principle be true or false. Usually, we do not know their truth values. Logic, however, does not tell us anything about contingent sentences (i.e., sentences which can be true or false—like “it is raining”—but which are neither tautologies nor contradictions). Among other things, logic allows us to identify tautologies (i.e., sentences which are necessary true, like “it is raining or it does not rain”) and contradictions (i.e., sentences which are necessary false, like “it is raining and it is not raining”). In common sense epistemic argumentation, however, tautologies or contradictions are usually neither claimed nor attacked, since the respective truth values are fixed *a priori*: there’s no point to argue about the truth of a tautology or about the falsehood of a contradiction. Thus, logic alone does not tell us anything informative about the world.

The situation is similar when it comes to inferences. Logic tells us whether epistemic sentences follow necessarily from the premises which are assumed to be true. Whether the assumed premises are actually true or not is again a question which cannot be answered based on *a priori* logical grounds. Again, more is needed than logic alone for the rational reconstruction of the epistemic argumentation.

2. PROBABILITY LOGIC ALONE IS NOT ENOUGH FOR THE EPISTEMOLOGY OF ARGUMENTATION
Recently, promising probabilistic approaches to argumentation have been proposed. Lumer’s approach is one of them. Probabilistic approaches offer a
more realistic rationality framework for modelling epistemic argumentation: contrary to logic, probability is defeasible (i.e., it allows for revisions in the light of new evidence). Moreover, beliefs in sentences are not restricted to “true” and “false”. Rather, probability allows for assigning degrees of belief.

However, while the enrichment of logic by probability offers a lot of epistemological plausibility, we are still in the same situation as with logic alone when it comes to epistemic justification: by the laws of probability theory, probability one needs to be assigned when we assess the probability of a tautology (however, the converse does not hold in general). Likewise, probability zero needs to be assigned when we assess the probability of a contradiction. However, what probabilities we assign to contingent sentences is not ruled out by probability theory alone (although the over-all assessment needs to be coherent, which is analogue to the requirement of logical consistency in rational argumentation).

Probability logic is about the coherent transmission of the premise probabilities to the conclusion. It does tell us how to assess the probability of the conclusion in the light of the premises but it does not tell us whether we have assigned the “right” probabilities to the premises beforehand. Thus, as in logic, probability logic does not tell us if the initial premise assessments were made correctly. In terms of Wittgenstein’s quote: therefore, the sentences of probability logic say nothing. (Note that this claim differs from Bruno de Finetti’s famous ontological claim about the subjective nature of probability: “Probability does not exist”, 1970/1974).

To build a bridge between “the world” and probability is therefore an important epistemological problem in general. Lumer’s paper sketches an interesting theory to build such a bridge in the context of epistemic argumentation theory. However, its sketchy nature calls for clarification. In the next sections, I briefly argue for the importance of the choice of a probability theory and of the choice of a statistical theory for such bridging theories. Thinking about these choices and their justification serves to help making Lumer’s theory more clear and more precise.

3. CHOOSING AN APPROPRIATE PROBABILITY THEORY

For any probabilistic or Bayesian epistemology, the choice of an appropriate probability theory is important. This is also the case when it comes to probabilistic analyses of argumentation. As an example, consider the probabilistic interpretation of conditionals. Here, it is crucial how zero-antecedent probabilities are interpreted. In standard approaches to probability, which define conditional probability by the fraction of the joint and the marginal probabilities (i.e., \( p(C|A) = \text{def.} \frac{p(A \& C)}{p(A)} \)), it is always assumed that the probability of the antecedent must not be equal to zero in order to avoid fractions over zero (i.e., it is assumed that \( p(A) > 0 \)). In the coherence approach to probability, however—which I advocate—conditional probabilities are conceived as primitive and are thus not defined by the fraction of the joint and the mar-
original probabilities. This avoids problems with zero-antecedent probabilities. Moreover, the definition of conditional probability has a major impact on the interpretation of conditionals. Conditionals (i.e., sentences of the form: if $A$, then $C$) play an important role in argumentative contexts. The interpretation of conditionals in terms of conditional probability ($p(C|A)$) has many advantages. For instance, it avoids paradoxes of the material conditional interpretation of conditionals: e.g., in coherence-based probability logic, the inference

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\text{from } p(C)=x, \text{ infer } 0 \leq p(C|A) \leq 1
\]

is probabilistically non-informative, since the unit interval ([0,1]) is the coherent assessment of the conclusion ($p(C|A)$) for any premise probability ($p(C)$) equal to $x$ (Pfeifer, 2014). This matches intuition. In standard approaches to probability, however, it holds that: if $p(C)=1$, then $p(C|A)$ must be equal 1 (or $p(C|A)$ is undefined if $p(A)=0$). Thus, in an argumentative context, if someone argues about “If $A$, then $C$” based on believing in $C$, it depends on the probabilistic background theory, whether the claimed conclusion holds or not. Of course in logic, the inference from $A$ to the material conditional “if $A$, then $C$” is logically valid. In both, in logic and in the standard approach to probability, we obtain counter-intuitive inferences when common sense sentences are instantiated for $A$ and $C$: based on “pope Francis is the current pope”, for example, it sounds counter-intuitive to claim that “If there is life on Mars, then pope Francis is the current pope”. In coherence-based probability logic, however, such inferences are blocked, since the $p(C|A)$ is not constrained even if $p(A)=1$, which corresponds to our intuition.

4. CHOOSING AN APPROPRIATE STATISTICAL THEORY

Bridges between “the world” and probability assessments are offered by statistical theories. Based on observed samples, statistical theories allow for drawing inferences about probabilistic relationships in the population. However, what kind of probabilistic relationships can be drawn depends on the choice of the statistical approach. As an example, let me point to two conceptually fundamental different statistical approaches. A standard approach to statistics in the social sciences is based on statistical significance testing. In a nutshell, statistical significance testing leads to decisions whether the null- or the alternative hypothesis should be accepted. The null hypothesis states that there is no difference (e.g., between the control group where a placebo was administered and the experimental group where a new drug was administered), whereas the alternative hypothesis states that there is a difference. The rationale for this choice is based on whether the data are sufficiently “improbable” based on the assumption that the null hypothesis is true: thus, it is based on the conditional probability of the data given the null hypothesis. The significance testing approach does not say anything about the probability of a given hypothesis. Rather, it delivers a decision. A fundamentally different approach is given by Bayesian statistics. It delivers a probability of hypothesis in the light of the evidence, i.e., it delivers the probability of a hypothesis given the data. Obviously, Bayesian statistics are fundamentally different from statistical theories which are based on signifi-
cance testing. I suggest to keep this difference in mind when realistic epistemic probabilistic theories of argumentation are constructed.

Since neither logic nor probability logic alone are enough for the epistemic justification of rational argumentation, bridges between “the world” and the claims about it are needed. Lumer’s contribution aims to build such a bridge. However, I argue that for building such a bridge it is important to be clear about the choice of an appropriate probability theory and an appropriate statistical theory. My choice would be to use coherence-based probability logic in combination with a well selected Bayesian statistical theory.

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