Abductive, causal, and counterfactual conditionals under incomplete probabilistic knowledge

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Introduction

Probabilistic truth table task in terms of probability logic:
Participants are presented with tasks containing the following premises:

$$\{p(A \wedge C) = x_1, p(A \wedge \neg C) = x_2, p(\neg A \wedge C) = x_3, p(\neg A \wedge \neg C) = x_4\}$$

and asked to infer their degree of belief in *If A, then C*. Based on their responses, the participants' interpretation of the conditional is given by:

Interpretation	Conclusion
Material conditional	$p(A \supset C) = x_1 + x_3 + x_4$
Conjunction	$p(A \wedge C) = x_1$
Biconditional	$p(A \equiv C) = x_1 + x_4$
Biconditional event	$p(C \vert\vert A) = x_1/(x_1 + x_2 + x_3)$
Conditional event	$p(C \vert A) = x_1/(x_1 + x_2)$

Observation:
Most people interpret their beliefs in conditionals by $p(C|A)$ even if $x_1, \ldots, x_4$ may be imprecise (Pfeifer, 2013) and the conditional is formulated as a counterfactual: *If A were the case, C would be the case* (see, e.g., Pfeifer & Stöckle-Schobel, 2015).

Research questions:

- How do people interpret causal (*if cause, then effect*) and abductive (*if effect, then cause*) conditionals?
- Are there response differences if they are formulated as indicative conditionals or as counterfactuals?
- How do people deal with imprecise probabilities?

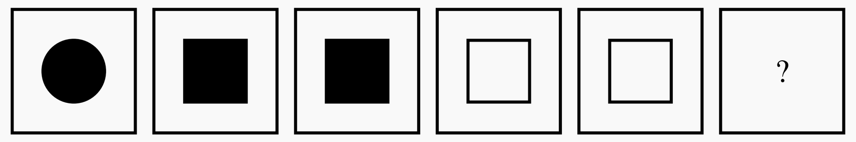
Method

- Participants: 80 Finnish university students.
- Material: 18 pen and paper tasks.
- Design: 2×2 between participants design:

	Type	Formulation	Sample
Condition 1	non-causal	indicative	$(n_1 = 20)$
Condition 2	non-causal	counterfactual	$(n_2 = 20)$
Condition 3	causal	counterfactual	$(n_3 = 20)$
Condition 4	abductive	counterfactual	$(n_4 = 20)$

Sample task 1 (non-causal, indicative):

Below are illustrated all the sides of a six-sided die. The sides of the die have two kinds of properties: color (*black* or *white*) and figure (*circle*, *triangle* or *square*).
Question mark means a covered side.



Imagine, that this die is placed in a cup. Then the cup is shaken randomly. Finally, the cup is placed on a table upside down, so that you cannot see which side of the die is facing upwards.

Question: How sure you can be, that the following sentence holds?

If the figure on the upward facing side of the die is a *circle*, **then** the figure is *black*.

Answer:

	at least	*at most*	
	☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6	☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6	how many
	☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6	☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6	out of how many

Sample task 2 (causal, counterfactual):

Here you see patient reports from medical studies concerning three new drugs. Each patient report shows the name of the new drug (*Zotarin*, *Xebutol* or *Raverat*) and its impact (*diminishing symptoms* or *no impact on symptoms*).
Question mark means a covered report.

Zotarin	*Xebutol*	*Xebutol*	*Xebutol*	*Xebutol*	?
no impact on symptoms	*no impact on symptoms*	*no impact on symptoms*	*diminishes symptoms*	*diminishes symptoms*	

Imagine a patient, who takes *Xebutol* and view the patient reports again.

Question: How sure you can be, that the following sentence holds?

If the patient were to take *Zotarin*, **then** this would have *no impact* on the symptoms.

From a probability logic point of view, sample tasks 1 and 2 are structured in the same way and imply the following predictions:

Interpretation	Predictions: at least	Predictions: at most	Hidden sides ignored
$p(\text{black} \mid \text{circle})$	1 out of 2	2 out of 2	no
$p(\text{black} \mid \text{circle})_{\bar{l}}$	1 out of 1	2 out of 2	lower bound
$p(\text{black} \mid \text{circle})_{\bar{u}}$	1 out of 2	1 out of 1	upper bound
$p(\text{black} \mid \text{circle})_{\overline{lu}}$	1 out of 1	1 out of 1	upper & lower bound
$p(\text{circle} \wedge \text{black})$	1 out of 6	2 out of 6	no
$p(\text{circle} \supset \text{black})$	5 out of 6	6 out of 6	no
$p(\text{circle} \equiv \text{black})$	3 out of 6	4 out of 6	no
$p(\text{circle} \vert\vert \text{black})$	1 out of 4	2 out of 4	no

Results

- No significant differences were observed among the four groups.
- Out of all responses 32.1% were consistent with conditional event responses $(\cdot|\cdot)$ and 29.9% with conjunction $(\cdot \wedge \cdot)$.
- Three types of half-way strategies related to the conditional probability responses $(p(\cdot|\cdot)_{\bar{l}}, p(\cdot|\cdot)_{\bar{u}}, p(\cdot|\cdot)_{\overline{lu}})$ were identified in the tasks with incomplete information. 51.5% of all responses are explained by the combination of the three half-way strategies and the $p(\cdot|\cdot)$ responses.
- The experimental results support the conditional probability interpretation of conditionals.

Discussion and concluding remarks

- Imprecise probabilities make the task more complex: half-way conditional event response strategies are used to reduce the cognitive load.
- Why is the belief in a counterfactual evaluated by the corresponding conditional probability? Formally (see, e.g. Gilio & Sanfilippo, 2013),

$$\overbrace{\text{Prevision}\ \underbrace{[(C|A)\,|\,\overset{\text{fact}}{\overbrace{\neg A}}]}_{\text{cond. random quantity}}}^{\text{belief in counterfactual}} = \overbrace{\text{Probability}\ \underbrace{(C|\ \overset{\text{assumed}}{\overbrace{A}}\ )}_{\text{cond.event}}}^{\text{belief in indicative conditional}}.$$

Conditional probability is the best predictor for how people interpret

- indicative and counterfactual non-causal conditionals, and
- abductive and causal counterfactuals.

Future work will focus on abductive and causal conditionals in indicative mood.

Acknowledgments and references

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