Probability Logic, Inferentialism, and Rationality*

David E. Over[0000–0002–0353–9381] [1], Angelo Gilio[2] ** [0000–0002–6898–7863], Niki Pfeifer[0000–0001–7129–5991] [3], and Giuseppe Sanfilippo[4] [0000–0002–0657–3833]

[1] Department of Psychology, Durham University, United Kingdom
david.over@durham.ac.uk
[2] Department SBAI, University of Rome "La Sapienza", Italy
angelo.gilio1948@gmail.com
[3] Department of Philosophy, University of Regensburg, Germany
niki.pfeifer@ur.de
[4] Department of Mathematics and Computer Science, University of Palermo, Italy
giuseppe.sanfilippo@unipa.it

Abstract. Probability logic, as we define it, is founded on the conditional probability hypothesis that the probability of a natural language conditional, $P(\textit{if } A \textit{ then } C)$, is $P(C|A)$, the conditional probability of C *given* A. This paper contrasts probability logic with narrow inferentialism and argues that non-constructive reasoning and independence conditionals are central to rationality. We discuss the psychology of conditionals and conditional bets, and illustrate the use of dilemma inferences in reasoning. Our approach provides a precise semantics for conditional reasoning and rational decision-making, highlighting the role of independence conditionals.

Keywords: Conditionals · Constructive Dilemma · Inferentialism · Non-constructive Reasoning · Probability Logic · Rationality

1 Introduction

Probability logic, as we will define it, is founded on the *conditional probability hypothesis* that the probability of the natural language conditional, $P(\textit{if } A \textit{ then } C)$, is $P(C|A)$, i.e., the conditional probability of C given A ([27]). This hypothesis can be justified by philosophical arguments and psychological experiments ([22,49]). A conditional that satisfies it has been called a *conditional event* ([18,19]) and a *suppositional conditional* ([49]). We stress that, in our approach, *if A then C* and *if not-A then C* can both be acceptable conditionals, as

---

* This paper will appear in the proceedings of the 4th International conference on Human and Artificial Rationalities (HAR 2025), in Lecture Notes in Computer Science. Cham: Springer. This is the penultimate version. Please do not quote without permission.

** Retired

happens when $P(C|A)$ and $P(C|not\text{-}A)$ are both high and perhaps equal. More informally, *if A then C* and *if not-A then C* can be acceptable when A and C are independent of each other. We will argue that one of the most important uses of conditionals is to derive and express facts about independence. It can be of significant utility and vital for rationality to learn that $P(\textit{if } A \textit{ then } C) = P(\textit{if not-}A \textit{ then } C)$, i.e., that $P(C|A) = P(C|not\text{-}A)$, as a result of conditional reasoning ([15]). The reasoning is Bayesian when the conditional is suppositional (see also [45] on probability logic and the new Bayesian paradigm in the psychology of reasoning).

Crupi and Iacona ([12]) have defined *inferentialism* as the view that an assertion of *if A then C* is acceptable if and only if C can be "inferred" from A. Going back to classical times, they have identified a number of logicians and philosophers who have proposed versions of "inferentialism" in this wide sense. But in this paper, we will interpret the term as the narrower claim that *if A then C* is acceptable if and only if C can be "inferred" from A but not also from *not-A*. This position directly implies that *if A then C* and *if not-A then C* cannot both be acceptable conditionals. Crupi and Iacona survey a number of "inferentialist" proposals that implicitly imply narrow inferentialism in our sense. For example, some of these state that *if A then C* is acceptable only if A increases the probability of C. It would be incoherent to claim that both A and *not-A* increased the probability of C. Douven et al. ([21]) explicitly adopt what we are calling narrow inferentialism. Douven and his collaborators have also worked hard to try to provide, not only arguments for their position in philosophy, but also experimental support for it in the psychology of reasoning (see [21] , for a summary). For these reasons, we will mainly focus on their proposals to make our points about narrow inferentialism.[1]

2 Probability Logic versus Inferentialism

We interpret a conditional *if A then C* as de Finetti's conditional event $C|A$. We then directly obtain the identity $P(\textit{if } A \textit{ then } C) = P(C|A)$, which has also been called *the Equation* because of its deep and far-reaching consequences ([22]). The logic of this suppositional conditional is what de Finetti ([19]) called "the logic of probability". We will refer to it simply as probability logic (see, e.g., [10,29,54]).

De Finetti's theory is the most general approach to the probabilistic treatment of uncertainty. It allows us to introduce probability assessments on arbitrary families of conditional events without requiring algebraic structures. Moreover, in this setting a conditional probability $P(C|A)$ is a primitive notion, and we do not need to define it as the ratio $\frac{P(AC)}{P(A)}$, which requires $P(A) > 0$; then, conditionals with antecedents of zero probability can be properly managed (see, e.g., [7,10,30,31,54]). For instance, let us consider an experiment in which an integer is chosen at random. We denote by E_n the event "the chosen number is

[1] In general, any account of *if A then C* which implies that $P(\textit{if } A \textit{ then } C) = P(C|A)$ could endorse our critique of narrow inferentialism.

the integer n", for $n = 1, 2, \ldots$. Since $P(E_n) = P(E_m)$ for every $n \neq m$, it follows that $P(E_n) = 0$ for all $n = 1, 2, \ldots$. In particular, $P(E_1) = P(E_2) = 0$, and hence $P(E_1 \vee E_2) = 0$. Then, the conditional probability $P(E_1 \mid (E_1 \vee E_2))$ cannot be evaluated by the ratio $\frac{P(E_1)}{P(E_1 \vee E_2)}$, because the latter expression reduces to $\frac{0}{0}$. However, the intuitive assessment $P(E_1 \mid (E_1 \vee E_2)) = \frac{1}{2}$ is allowed in de Finetti's subjective probability theory because this evaluation is a coherent extension of the assessment $P(E_1) = P(E_2) = 0$.

A relevant application of de Finetti's coherence-based approach in the context of probabilistic nonmonotonic reasoning is the well-known System P ([1,2,8,26,30], for psychological studies on System P see [17,51,52]), with an extension to conjunctions and disjunctions of conditionals ([27,61]). A related approach to compound conditionals, restricted to the case where the antecedents have positive probability, has been taken by many authors (see, e.g., [36,42,66,70]). A probabilistic study of Boolean algebra of conditionals, in the setting of coherence, has been given in [24,25].

Probability logic has *non-constructive* aspects, like classical logic. Most fundamentally, *A or not-A*, the non-constructive *law of excluded middle*, is a logical truth in probability logic, and the *conditional law of the excluded middle* is a logical truth in probability logic:

(CLEM) *if A then C or if A then not-C*.

The validity of the inference rules of *one-premise* and *two premise conditional dilemma* can also be shown to be valid (provided that $A \,\&\, C$ and *not-A* $\&\, C$ are not contradictions) in probability logic ([28]):

(1CD) From the premise *if A then C and if not-A then C* infer C.

(2CD) From the premises *if A then C* and *if not-A then C* infer C.

These inferences, (1CD)[2] and (2CD), are non-constructive when *A or not-A* is a logical truth, and it is not known whether A holds or *not-A* holds. There is an unfortunate terminological difference in the literature that could cause confusion at this point. Copi and Cohen ([11]) describe a more general inference rule than (1CD) or (2CD) which they call "constructive dilemma". But in this *rhetorical* use, a "constructive" dilemma is not opposed to a logically "non-constructive" one, but rather to a rhetorically "destructive" dilemma, which has negations in it and is sometimes used to attack an opponent.

We will use the only logical distinction between "constructive" and "non-constructive" in this paper. Douven et al. ([21]) argue for what appears to be the most strongly *constructive*, in our sense, version of inferentialism in the literature. For them, *if A then C* is true as a "standard" conditional when an argument—deductive, inductive, or abductive—can be constructed from A to C,

[2] Notice that in the conditional dilemma (1CD) the premise is the conjunction of two conditionals. A coherence-based theory of compound and iterated conditionals in the setting of conditional random quantities has been developed in [27,32,33,34,61].

possibly with background information. In this argument A has to be "pivotal", which they take to imply that there cannot be such an argument as well from *not-A* to C. They add that *if A then C* is false when there is such an argument from A to *not-C* and is indeterminate otherwise. They have not clarified this definition by developing a formal semantics and logic for their theory. Denying the conjunctive premise of (1CD) is sometimes referred to as "Aristotle's Second Thesis" in writings on *connexive logic*, which is supposed to be a logic about some tight connection between A and C in *if A then C* ([53,55,59]). It is unclear how close a Douven et al. logic would be to a connexive logic, but their theory implies the rejection of (CLEM) as a necessary truth, and the unacceptability of the premise of (1CD) and of at least one of the premises of (2CD) for "normal" or "standard" conditionals. They contend that *if A then C* is not "normal" or "standard" when C holds whether or not A (see also [20, pp. 109–110]). Let us consider some examples of the application of (1CD) or (2CD) to illustrate our points.

First, there is the *blocks example* ([39]). Suppose we are told that there is a stack of blocks, some green, and some not green, and that the second block in the stack is green, and the fourth not green. Is there a green block directly on top of a non-green block in this stack? We could use (1CD) or (2CD) to answer this question, reasoning that, if the third block is green, the answer is "Yes", and that, if the third block is not green, the answer is "Yes". Therefore, the answer to the question is "Yes" (see [68], for an experiment on how difficult this problem is for participants).

Second, there is the *free speech example* from Mill ([43, Ch. II]), *On Liberty.* Suppose that A is an opinion which we have "silenced" because we believe *not-A*. Mill's conditionals are in relatively complex Victorian prose, but he basically argued that, if A is true, then there is a loss in utility. By not hearing about A, we lose the chance to believe the truth, and this will generally harm us. But if A is not true, then we also lose utility. In this case, we lose the challenge to our belief in *not-A* and the opportunity to deepen our justification of it, and this will generally harm us. Therefore, Mill concluded, we lose utility in any case in which A is "silenced", whether A is true or not. Crupi and Iacona ([12]) have Mill down as an "inferentialist", but they do not refer to Mill ([43]). They only refer to Mill ([44]), his book on logic, and even there, Mill wrote that *if A then C* affirms that C is inferable from A. He did not add that C must not be inferable from *not-A* as well. He was evidently not an inferentialist in our narrow sense.

The abstract blocks example brings out the non-constructive essence of (2CD) most clearly. No stack of blocks is constructed in any sense for this problem. Mill's argument makes the point that conditional dilemmas can be of considerable significance: it is one of the most famous in the history of political theory. Copi and Cohen ([11]) do not give it as an example, but they list many real-world examples of (1CD) or (2CD) and of their more general form. They conclude that "... the dilemma is perhaps the most powerful instrument of persuasion ever devised". That may well be too strong. Psychological research arguably demonstrates that there is no "powerful" way to remove myside bias ([49,67]). But we do not think it can be denied that dilemma inferences have a valuable role in reasoning.

Our third example is simpler and more concrete. For it, we will suppose that a teacher asserts the following two conditionals about a highly intelligent student who has attended all the classes and done all the homework:

(1) If Jane revises, she will pass the exam.

(2) If Jane does not revise, she will pass the exam.

Trusting the teacher, we use (2CD) to infer that Jane will pass the exam. In contrast, Douven et al. ([21]) would deny that (1) and (2) could both hold for "standard" or "normal" conditionals, to which their theory applies. They would call (1) and (2) "abnormal" conditionals in the context we describe, with "missing links" in them between the antecedents and consequent. They have a *whether-or-not test* for these "non-standard" cases. If we can say "whether or not A, C", then *if A then C*, and *if not-A then C*, are "non-standard", "abnormal", and "missing link" conditionals. As we can say, "whether or not Jane revises, she will pass", and that "whether or not block three is green, the answer to the blocks example is Yes", and that "whether or not A is true, utility is lost" in the free speech example, the conditionals in all these examples would be classified by Douven et al. as "non-standard" and "abnormal". They would conclude that their theory of "standard" conditionals does not have to apply to them. Douven et al. ([21]) did very briefly and tentatively suggest a semantics for "non-standard" conditionals beyond their main theory, but they claimed that the use of these conditionals is a peripheral and unimportant aspect of communication and reasoning.

Douven et al. ([21]) even refer to "normal" people as those who use "normal" conditionals, but Lassiter ([37]) has pointed out that "... the notion of a 'normal' use does not have a theoretical interpretation in natural language semantics". What constructive inferentialists are claiming, beyond the "abnormal" and "non-standard" terms, is that "if" is ambiguous, and that one of its meanings, the one that does not fit their theory, is of little or no importance in language use and conditional reasoning.

Our view is completely different. We do distinguish between *dependence* conditionals, for which $P(C|A) > P(C|not\text{-}A)$, and *independence* conditionals, for which $P(C|A) = P(C|not\text{-}A)$, such as (1) and (2). But this distinction is pragmatic in our approach ([15]). In it, there is only one conditional in natural language, a suppositional conditional, the conditional event, which is sometimes a dependence conditional, and sometimes an independence conditional, depending on the context. We argue that uses of independence conditionals, like those in our examples, are far from a peripheral and unimportant part of human cognition. Knowledge of independence, whether conditional or not, can be of great value in reasoning and decision making and is often a necessary condition for rationality ([15,47,49]). This important knowledge is often conveyed by uses of independence conditionals. Dilemma inferences, (1CD) or (2CD), are sometimes central to deriving it, as illustrated in our examples, although depending on the exact value of $P(C|A)$ and of $P(C|not\text{-}A)$, they may not always establish that A and C are precisely independent.

Suppose that (2) is used as a *concessive* ("even if") conditional in the context we have described, with "Even if" replacing "If" at the front of it. In such contexts, concessive conditionals are independence conditionals that convey useful information about independence, in this example that whether Jane passes the exam is independent of whether she revises. For (2) to do this, (1) has to be at least implicitly in the background as an independence conditional as well in this context. It cannot be the case, for instance, that revising seriously lowers the probability of passing, say, because of a mistake in the revision materials. With (1) at least implicit, and (1) and (2) highly probable, (1CD) or (2CD) can be used to infer with confidence that Jane will pass the exam. Uses of concessive conditionals generate many (often implicit) independence conditionals ([15]), with (1CD) or (2CD) arguments in underlying support of the conclusions drawn from them. [3]

We can reinforce our point about concessives and demonstrate the relation between the dilemma inferences and Bayesian reasoning with yet another example. Consider a pregnancy test that becomes increasingly unreliable, producing more and more false positives, as it gets close to its expiry date, when it gives positive results no matter what, and the following conditionals:

(3) If Jane is pregnant, the test will be positive.

(4) If Jane is not pregnant, the test will be positive.

Letting H (the hypothesis) and E (the evidence) be the antecedent and consequent of (3), we have in our de Finetti theory $P(\textit{if } H \textit{ then } E) > P(\textit{if not-}H \textit{ then } E)$ before the expiry date and $P(\textit{if } H \textit{ then } E) = P(\textit{if not-}H \textit{ then } E)$ after that date. We can use Bayesian reasoning before the date to infer that $P(\textit{if } E \textit{ then } H) > P(H)$, i.e., a positive test result increases the probability that Jane is pregnant. As the expiry date approaches, and $P(\textit{if not-}H \textit{ then } E)$ increases more and more, it might become more and more pragmatically appropriate to assert (4), perhaps with "Even if" replacing "If". After that date, $P(\textit{if } E \textit{ then } H) = P(H)$ by Bayesian reasoning, i.e., the test result and the pregnancy are independent. Knowledge of this fact would be necessary for Jane's rational decision making. When (3) and (4) are both high, Jane can of course use a dilemma inference to infer with confidence that the test will be positive, whether or not she is pregnant. In our approach, we can give a smooth and unbroken account of this reasoning up to and through the expiry date (see [35] on Bayes' theorem).

In contrast, Douven et al. ([21]) inferentialism implies that (3) and (4) become "abnormal" and "non-standard" conditionals when the expiry date arrives and the pregnancy and the test result become independent. Indeed, at that point, (3) becomes, for them, essentially the same as a conditional like "If Jane is pregnant

[3] In a concessive conditional, "even" is applied to a suppositional "if". Neither conditional is abnormal. See Bennett ([5]) for an analysis of concessives that has influenced ours, and Lycan ([40]) and Crupi and Iacona ([13]) for other analyses.

then Oxford is in England" (see the next section on "Walrus conditionals" like this one), and their main theory does not apply. Moreover, they would have to add that it is rare for conditionals to be used in Bayesian reasoning to infer independence. We would simply challenge them to provide a convincing account of how people do use reasoning to learn about independence ([15,47,49]).

3 Conditionals in the Psychology of Reasoning

Psychologists of reasoning have not extensively studied non-constructive reasoning, identified as such, but they have tested the Equation mostly via the Probabilistic Truth Table Task since Evans, Handley, and Over ([23]). Over and Evans ([49]) summarise this research by saying that it has confirmed that $P(\textit{if } A \textit{ then } C) = P(C|A)$ for pragmatically acceptable *if A then C* (significant results are in, for examples, [46,50,63,64]). But Edgington ([22]) predicted long ago the Equation would fail for pragmatically unacceptable conditionals. Her example was:

(5) If Napoleon is dead, Oxford is in England.

Over and Evans ([49, 5.2]) call examples like (5) *Walrus conditionals* in reference to the Walrus in Lewis Carroll's nonsense poem "The Walrus and the carpenter", who wants to talk about a pragmatically bizarre list of unrelated topics: shoes, ships, sealing wax, cabbages, and kings. Walrus conditionals, by our definition, refer to pragmatically unrelated topics, like cabbages and kings, in their antecedents and consequents (see [6] for relevant work on topicality).

Skovgaard-Olsen, Singmann, & Klauer ([65]) compared pragmatically acceptable dependence conditionals such as,

(6) If Mark presses the power button on his TV, then the TV will be turned on,

with independence conditionals that were also Walrus conditionals,

(7) If Mark is wearing socks, then his TV will be working.

Confirming Edgington, Skovgaard-Olsen et al. discovered that the Equation was supported for conditionals like (6) but not for Walrus conditionals like (7) (see also [41,69]). This finding was called the *relevance effect*, because the antecedent of (6), but not of (7), is "relevant" in some sense to its consequent. It was certainly of value, but it was based on a confound. It compared pragmatically acceptable dependence conditionals, like (6), with pragmatically unacceptable independence conditionals, like (7), and not with pragmatically acceptable independence conditionals, like (1) and (2), or (3) and (4) beyond the expiry date of the pregnancy test ([49, 5.2]).

Douven et al. ([21]) argue that the experiments of Douven and his collaborators in the psychology of reasoning confirm their theory, but these studies contain the confound of comparing pragmatically acceptable dependence conditionals with pragmatically unacceptable independence conditionals and have other problems (see [15,16,48]). There is also Zhan and Wang ([71]) on how the relevance effect could be confounded by boundary cases. We will not repeat these critiques here. But we will add that we favour, on theoretical and experimental grounds, a pragmatic account of the findings in Douven et al. ([21]) and similar experiments ([9,14,37,38]).

We will, however, quickly use (7) to illustrate a problem with the inferentialist claim in Douven ([20, p. 108]) that *if A then C* is "acceptable" if and only if $P(C|A)$ is above some threshold and $P(C|A) > P(C)$ (which implies that $P(C|A) > P(C|not\text{-}A)$). Let S be the antecedent and W the consequent of (7). We would naturally think of an example in which $P(S)$ and $P(W)$ are high, and $P(W|S) = P(W|not\text{-}S)$. The de Finetti ([18]) *normal form* for

$$\textit{if } S \textit{ then } W \text{ is } \textit{if } S \textit{ then } S \,\&\, W.$$

Now in a natural example, we can make $P(W|S) = P(S \,\&\, W|S)$ above any threshold less than 1, and it follows that $P((S \,\&\, W)|S) > P((S \,\&\, W)|not\text{-}S)$, and $P((S \,\&\, W)|S) > P(S \,\&\, W)$, as

$$P(S \,\&\, W) = P(S)P(W|S) = P(S)P((S \,\&\, W)|S).$$

However, *if S then S & W* is no more intuitively acceptable than *if S then W*, which is the Walrus conditional (7). Over and Evans ([49]) use a similar argument to show that $P(C|A) > P(C|not\text{-}A)$ cannot be a necessary condition for a conditional to be "acceptable".

Furthermore, Pfeifer ([50]) pointed out that in the Probabilistic Truth Table Tasks, where participants are asked to assess their degree of belief in a conditional of the form *if A then C*, information about both $P(C|A)$ and $P(C|not\text{-}A)$ is available to the participants, and so measures of connections between both conditionals, like $\Delta p =_{def.} P(C|A) - P(C|not\text{-}A)$, can be computed. However, the key finding is that most participants respond with $P(C|A)$ *independently* of whether $P(C|A)$ is less, equal or greater than $P(C|not\text{-}A)$. Interestingly, this result is also independent of whether *non-causal* task material is used (e.g., when A is about a geometric figure and C is about a colour), or whether the task material is about clear *causal* or *abductive* connections between A and C ([56,57]). These results are incompatible with the inferentialists' claim that acceptance of *if A then C* requires at least a positive Δp (i.e., $P(C|A) > P(C|not\text{-}A)$).

In brief, constructive inferentialism does not appear to give us necessary or sufficient conditions for a conditional to be "acceptable", "normal", or "standard".

4 Conditional Bets

In our de Finetti approach, the assertion of a conditional is intimately related to a conditional bet, and there is experimental evidence to support this link ([4,58]). Imagine someone makes the following conditional bet:

(8) If Dobbin runs in the race, then I bet she will come first.

It is highly intuitive that, if Dobbin runs in the race and comes first, the bet (8) is won. If Dobbin runs in the race and does not come first, the bet is lost. If Dobbin does not run in the race, the bet is void, and no money changes hands. Assume Dobbin does run and comes first. We doubt that bettors would be impressed by an inferentialist who claimed that the bet was "lost", and no money should be paid out, because there was an extremely strong inductive inference, based on bad performance in the past, to the conclusion that Dobbin would not come first. Arguments like that are irrelevant in our account, as they are at racecourses and bookies.

A different kind of example does seem to go the other way. Suppose Jane loses a ring, and John says to her:

(9) If I look in your handbag, I bet you 50 Euros I will find it there.

Jane looks in her handbag and sees that the ring is definitely not there, so she demands 50 Euros from John, claiming that (9) is lost. But he replies that (9) is void because he did not look in the bag. One inferentialist position would be that John should pay up, as Jane has demonstrated that there could not be a strong argument from John's looking in the bag to his finding the ring there. But we would argue that John has suggested pragmatically that the ring is in the bag. That can be taken as his bet, so implying that he should pay Jane the 50 Euros.

For a more complex example, suppose Jane and John own a horse, and he makes this bet with her:

(10) If I enter our horse in the race, I bet you 50 Euros she will win.

Jane enters the horse in the race, and it loses badly. She demands 50 Euros from John, but he not only points out that he did not enter the horse in the race. He adds that he would only have done so if it had recovered from being lame. In this example, it is more intuitive, we would suggest, that (10) is void. There are a number of differences between (9) and (10), but the way intuitions about void bets can move about like this suggests to us that pragmatics should be applied to explain them. Controlled psychological experiments in the psychology of reasoning will have to be run on cases like (9) and (10) to test possible explanations of them.

We also hold that the use of an indicative conditional, *if A then C*, can convey pragmatically that C is inferable from A but not from *not-A*. In other contexts, it is suggested, or explicitly shown in the use of (1CD) or (2CD), that C follows from both A and *not-A*. We will not try to justify these psychological hypotheses here. But we do argue that a big advantage of our approach over constructive inferentialism is that we have a precise underlying semantics and logic for our theory, probability logic.

5 Concluding Remarks

Dilemma inferences are used to state many paradoxes, or apparent ones, and so are "dilemmas" in that special sense. For example, there is Russell's "paradox" of a male barber in a village who shaves men in the village if and only if they do not shave themselves ([60]). If this barber shaves himself (A), then he shaves himself and does not shave himself (A & *not*-A). If he does not shave himself (*not*-A), then he shaves himself and does not shave himself (A & *not*-A). Thus, the barber shaves himself and does not shave himself (A & *not*-A). The final step in this reasoning can be a *reductio*, concluding that such a barber does not exist. However, we observe that the probabilistic validity of (1CD) and (2CD) is not applicable in this particular case, since the conclusion of Russell's "paradox" is a contradiction. Sometimes people do seem to like to amuse themselves with this kind of reasoning in natural language, but it is far more seriously used in some profound proofs in logic, and a full analysis of it would lead us far from the topic of this paper.

The *sure-thing principle* in decision making is analogous to the dilemma inferences, (1CD) and (2CD), in reasoning. Savage ([62]) stated this principle to help guide his axiomatization of subjective expected utility for decision theory and his account of rationality. He expressed it in an instance of the following form. If an agent would prefer to perform action X if Y and X if *not*-Y, then he should perform X before he learns whether Y holds or not. The principle has "would" in its conditionals, includes a temporal element, and ends with a deontic "should" conclusion. These aspects would complicate a discussion of its relation to the dilemma inferences and take too many words in this paper. Savage and de Finetti influenced each other in their development of the subjective theory of probability, but extending our account of the conditional event and logical coherence to the rationality of the sure-thing principle and decision making is beyond the scope of this paper. We will just point out that the sure-thing conditionals pass the whether-or-not test and must be classified as "abnormal" and "non-standard" by narrow inferentialists and placed beyond their main theory. Psychological studies of the principle have found that people do not always conform to it ([3]), but its conditionals can hardly be called "abnormal" or unimportant on that basis. Participants in experiments sometimes commit fallacies in "normal" reasoning by any reasonable definition of this term ([49]).

The sure-thing principle and the dilemma inferences contain the independence conditional, which can be used to reason about independence relations. Douven et al. inferentialism unjustly gives the independence conditional "a bad name to hang it", labelling it as "abnormal". In contrast, the conditional event in probability logic can be used as a dependence conditional in some contexts and an independence conditional in others, and the dilemma inferences are p-valid in probability logic and can be of great value. We use probability logic and its conditional event to acquire knowledge of independence which is essential for rationality.

Acknowledgments. We would like to thank Dan Lassiter and an anonymous reviewer for very helpful comments. G. Sanfilippo acknowledges support by the FFR project of University of Palermo, Italy, by INdAM/GNAMPA research group, and by the PRIN 2022 project "Models for dynamic reasoning under partial knowledge to make interpretable decisions" (Project number: 2022AP3B3B, CUP Master: J53D23004340006, CUP: B53D23009860006) funded by the European Union – Next Generation EU.

References

1. Adams, E.W.: The logic of conditionals. Reidel, Dordrecht (1975)
2. Adams, E.W.: A Primer of Probability Logic. CLSI Publications, Stanford (1998)
3. Bagassi, M., Macchi, L.: Pragmatic approach to decision making under uncertainty: The case of the disjunction effect. Thinking & Reasoning **12**(3), 329–350 (2006)
4. Baratgin, J., Politzer, G., Over, D.E., Takahashi, T.: The psychology of uncertainty and three-valued truth tables. Frontiers in Psychology **9**, 1–18 (2018)
5. Bennett, J.: A Philosophical Guide to Conditionals. Clarendon Press, Oxford (2003)
6. Berto, F.: Topics of Thought: The Logic of Knowledge, Belief, Imagination. Oxford University Press, Oxford (2022)
7. Biazzo, V., Gilio, A.: A generalization of the fundamental theorem of de Finetti for imprecise conditional probability assessments. International Journal of Approximate Reasoning **24**(2), 251–272 (2000)
8. Biazzo, V., Gilio, A., Lukasiewicz, T., Sanfilippo, G.: Probabilistic logic under coherence, model-theoretic probabilistic logic, and default reasoning in System P. Journal of Applied Non-Classical Logics **12**(2), 189–213 (2002)
9. Bourlier, M., Jacquet, B., Lassiter, D., Baratgin, J.: Coherence, not conditional meaning, accounts for the relevance effect. Frontiers in Psychology **14**, 1–17 (2023)
10. Coletti, G., Scozzafava, R.: Probabilistic logic in a coherent setting. Kluwer, Dordrecht (2002)
11. Copi, I.M., Cohen, C.: Introduction to Logic. Macmillan, New York, 8th edn. (1990)
12. Crupi, V., Iacona, A.: Conditionals: Inferentialism explicated. Erkenntnis **90**, 2823–2854 (2024)
13. Crupi, V., Iacona, A.: On the logical form of concessive conditionals. Journal of Philosophical Logic **51**, 633–651 (2022)
14. Cruz, N., Over, D., Oaksford, M., Baratgin, J.: Centering and the meaning of conditionals. In: Proceedings of the 38th Annual Conference of the Cognitive Science Society, pp. 1104–1109. Cognitive Science Society, Austin, US (2016)
15. Cruz, N., Over, D.E.: Independence conditionals. In: Kaufmann, S., Over, D.E., Sharma, G. (eds.) Conditionals–Logic, linguistics and psychology, pp. 223–233. Palgrave Macmillan, London (2023)
16. Cruz, N., Over, D.E.: From de Finetti's Three Values to Conditional Probabilities in the Psychology of Reasoning. In: Égré, P., Rossi, L. (eds.) Handbook of Trivalent Logics. Cambridge: MIT Press (To appear). `https://doi.org/10.31234/osf.io/bxjt2_v1`
17. Da Silva Neves, R., Bonnefon, J.F., Raufaste, E.: An empirical test of patterns for nonmonotonic inference. Annals of Mathematics and Artificial Intelligence **34**, 107–130 (2002)
18. de Finetti, B.: Foresight: Its logical laws, its subjective sources. In: Studies in subjective probability, pp. 55–118. Krieger, Huntington (1980), english translation of "La Prévision: Ses Lois Logiques, Ses Sources Subjectives", Annales de l'Institut Henri Poincaré, 17:1-68 (1937)

19. de Finetti, B.: The logic of probability. Philosophical Studies **77**, 181–190 (1995), english translation of "La Logique de la Probabilité". In Actes du Congrès International de Philosophie Scientifique, Paris, 1935, pages IV 1–IV 9. Hermann et C.ie, Paris, (1936)
20. Douven, I.: The Epistemology of Indicative Conditionals. Cambridge University Press, Cambridge (2016)
21. Douven, I., Elqayam, S., Krzyzanowska, K.: Inferentialism: A manifesto. In: Kaufmann, S., Over, D.E., Sharma, G. (eds.) Conditionals—Logic, linguistics and psychology, pp. 175–221. Palgrave Macmillan, London (2023)
22. Edgington, D.: On conditionals. Mind **104**, 235–329 (1995)
23. Evans, J.S.B.T., Handley, S.J., Over, D.E.: Conditionals and conditional probability. Journal of Experimental Psychology: Learning Memory and Cognition **29**, 321–335 (2003)
24. Flaminio, T., Gilio, A., Godo, L., Sanfilippo, G.: Compound Conditionals as Random Quantities and Boolean Algebras. In: Kern-Isberner, G., Lakemeyer, G., Meyer, T. (eds.) Proceedings of the 19th International Conference on Principles of Knowledge Representation and Reasoning, KR 2022, Haifa, Israel. July 31 - August 5, 2022. pp. 141–151 (2022), https://proceedings.kr.org/2022/15/
25. Flaminio, T., Gilio, A., Godo, L., Sanfilippo, G.: On conditional probabilities and their canonical extensions to Boolean algebras of compound conditionals. Int. J. Approx. Reason. **159**, 108943 (2023). https://doi.org/10.1016/j.ijar.2023.108943
26. Gilio, A.: Probabilistic reasoning under coherence in System P. Annals of Mathematics and Artificial Intelligence **34**, 5–34 (2002)
27. Gilio, A., Over, D.E., Pfeifer, N., Sanfilippo, G.: On trivalent logics, probabilistically weak deduction theorems, and a general import-export principle. Artificial Intelligence **337C**, 104229 (2024). https://doi.org/https://doi.org/10.1016/j.artint.2024.104229
28. Gilio, A., Over, D.E., Pfeifer, N., Sanfilippo, G.: Constructive inferentialism versus non-constructive probability logic. (in preparation)
29. Gilio, A., Pfeifer, N., Sanfilippo, G.: Transitivity in coherence-based probability logic. Journal of Applied Logic **14**, 46–64 (2016)
30. Gilio, A., Sanfilippo, G.: Probabilistic entailment in the setting of coherence: The role of quasi conjunction and inclusion relation. International Journal of Approximate Reasoning **54**(4), 513–525 (2013). https://doi.org/10.1016/j.ijar.2012.11.001
31. Gilio, A., Sanfilippo, G.: Quasi conjunction, quasi disjunction, t-norms and t-conorms: Probabilistic aspects. Information Sciences **245**, 146–167 (2013). https://doi.org/10.1016/j.ins.2013.03.019
32. Gilio, A., Sanfilippo, G.: Conditional random quantities and compounds of conditionals. Studia Logica **102**(4), 709–729 (2014). https://doi.org/10.1007/s11225-013-9511-6
33. Gilio, A., Sanfilippo, G.: Generalized logical operations among conditional events. Applied Intelligence **49**(1), 79–102 (Jan 2019). https://doi.org/10.1007/s10489-018-1229-8
34. Gilio, A., Sanfilippo, G.: On compound and iterated conditionals. Argumenta **6**(2), 241–266 (2021). https://doi.org/10.14275/2465-2334/202112.gil, https://www.argumenta.org/article/compound-iterated-conditionals/
35. Joyce, J.: Bayes' theorem. In: Zalta, E.N. (ed.) The Stanford Encyclopedia of Philosophy (Fall 2021 Edition), https://plato.stanford.edu/archives/fall2021/entries/bayes-theorem/

36. Kaufmann, S.: Conditionals right and left: Probabilities for the whole family. Journal of Philosophical Logic **38**, 1–53 (2009). https://doi.org/10.1007/s10992-008-9088-0
37. Lassiter, D.: Decomposing relevance in conditionals. Mind & Language **38**(3), 644–668 (2023)
38. Lassiter, D., Li, C.: Pragmatic intrusion in probability judgment: The case of conditionals. In: Proceedings of the 46th Annual Conference of the Cognitive Science Society. pp. 1464–1471 (2024)
39. Levesque, H.J.: Making believers out of computers. Artificial Intelligence **30**, 81–108 (1986)
40. Lycan, W.G.: Real conditionals. Oxford University Press, Oxford (2001)
41. Matalon, B.: Étude génétique de l'implication. In: Piaget, J. (ed.) Études d'épistémologie génétique XVI. Presses Universitaires de France, Paris (1962)
42. McGee, V.: Conditional probabilities and compounds of conditionals. Philosophical Review **98**(4), 485–541 (1989). https://doi.org/http://dx.doi.org/10.2307/2185116
43. Mill, J.S.: On Liberty. John W. Parker and Son, London (1859)
44. Mill, J.S.: A System of Logic, Ratiocinative and Inductive. Harper and Brothers, New York, 8th edn. (1882)
45. Oaksford, M., Chater, N.: New paradigms in the psychology of reasoning. Annual Review of Psychology **71**, 305–330 (2020)
46. Oberauer, K., Weidenfeld, A., Fischer, K.: What makes us believe a conditional? Thinking & Reasoning **13**(4), 340–369 (2007)
47. Over, D.E.: Independence and rationality. The Reasoner **17**(6) (2023), focus issue on the Handbook of Rationality, edited by Marus Knauff & Wolfgang Spohn
48. Over, D.E., Cruz, N.: Indicative and counterfactual conditionals in the psychology of reasoning. In: Kaufmann, S., Over, D.E., Sharma, G. (eds.) Conditionals—Logic, linguistics and psychology, pp. 139–174. Palgrave Macmillan, London (2023)
49. Over, D.E., Evans, J.S.B.T.: Human Reasoning. Cambridge University Press, Cambridge (2024)
50. Pfeifer, N.: The logic and pragmatics of conditionals under uncertainty: a mental probability logic perspective. In: Kaufmann, S., Over, D.E., Sharma, G. (eds.) Conditionals—Logic, linguistics and psychology, pp. 73–102. Palgrave Macmillan, London (2023)
51. Pfeifer, N., Kleiter, G.D.: Coherence and nonmonotonicity in human reasoning. Synthese **146**(1-2), 93–109 (2005)
52. Pfeifer, N., Kleiter, G.D.: Framing human inference by coherence based probability logic. Journal of Applied Logic **7**, 206–217 (2009)
53. Pfeifer, N., Sanfilippo, G.: Connexive logic, probabilistic default reasoning, and compound conditionals. Studia Logica **112**, 167–206 (2024), https://doi.org/10.1007/s11225-023-10054-5
54. Pfeifer, N., Sanfilippo, G.: Probability propagation rules for Aristotelian syllogisms. Annals of Pure and Applied Logic **175**(9) (2024). https://doi.org/https://doi.org/10.1016/j.apal.2023.103340
55. Pfeifer, N., Schöppl, L.: Experimental philosophy of connexivity. In: 60 years of connexive logic, pp. 201–223. Springer (Trends in Logic), Cham (2025), preprint: https://philpapers.org/rec/PFEEPO
56. Pfeifer, N., Stöckle-Schobel, R.: Uncertain conditionals and counterfactuals in (non-)causal settings. In: Airenti, G., Bara, B.G., G., S. (eds.) Proceedings of

the EuroAsianPacific Joint Conference on Cognitive Science (4^{th} European Conference on Cognitive Science; 10^{th} International Conference on Cognitive Science). vol. 1419, pp. 651–656. CEUR Workshop Proceedings, Aachen (2015), http://ceur-ws.org/Vol-1419/paper0108.pdf
57. Pfeifer, N., Tulkki, L.: Abductive, causal, and counterfactual conditionals under incomplete probabilistic knowledge. In: Gunzelmann, G., Howes, A., Tenbrink, T., Davelaar, E. (eds.) Proceedings of the 39^{th} Cognitive Science Society Meeting. pp. 2888–2893. The Cognitive Science Society, Austin, TX (2017)
58. Politzer, G., Over, D.E., Baratgin, J.: Betting on conditionals. Thinking & Reasoning **16**, 172–197 (2010)
59. Rott, H.: Conditionals, support and connexivity. In: Omori, H., Wansing, H. (eds.) 60 Years of Connexive Logic, Trends in Logic, vol. 65. Springer, Cham (2025). https://doi.org/10.1007/978-3-031-82994-9_7
60. Russell, B.: The philosophy of logical atomism. The Monist **29**(3), 345–380 (1919)
61. Sanfilippo, G., Gilio, A., Over, D.E., Pfeifer, N.: Probabilities of conditionals and previsions of iterated conditionals. International Journal of Approximate Reasoning **121**, 150 – 173 (2020). https://doi.org/10.1016/j.ijar.2020.03.001
62. Savage, L.J.: The Foundations of Statistics. Wiley, New York (1954)
63. Singmann, H., Klauer, K.C., Over, D.E.: New normative standards of conditional reasoning and the dual-source model. Frontiers in Psychology **5**, 316 (2014)
64. Skovgaard-Olsen, N., Kellen, D., Krahl, H., Klauer, K.C.: Relevance differently affects the truth, acceptability, and probability evaluations of 'and', 'but', 'therefore', and 'if then'. Thinking & Reasoning **23**, 449–482 (2017)
65. Skovgaard-Olsen, N., Singmann, H., Klauer, K.C.: The relevance effect and conditionals. Cognition **150**, 26–36 (2016)
66. Stalnaker, R.C.: Probability and conditionals. Philosophy of Science **37**(1), 64–80 (1970)
67. Stanovich, K.E.: The Bias That Divides Us: The Science and Politics of Myside Thinking. MIT Press, Cambridge, Mass. (2021)
68. Toplak, M.E., Stanovich, K.E.: The domain specificity and generality of disjunctive reasoning: Searching for a generalizable critical thinking skill. Journal of Educational Psychology **94**(1), 197–209 (2002). https://doi.org/10.1037/0022-0663.94.1.197
69. Vidal, M., Baratgin, J.: A psychological study of unconnected conditionals. Journal of Cognitive Psychology **29**(6), 769–781 (2017)
70. Wójtowicz, K., Wójtowicz, A.: Probabilities of conditionals via labeled markov graphs. Erkenntnis (2025). https://doi.org/10.1007/s10670-024-00896-3
71. Zhan, L., Wang, M.: Probabilities of conditionals: The relevance effect might be confounded by the existence of boundary cases. Psychonomic Bulletin & Review (in press)