Introduction to Stable Homotopy Theory
Exercise Sheet 8

1. Let X be a spectrum and n be an integer. Show that every class in $\pi_*(X/n)$ is killed by n^2.

2. Show that every connective spectrum is $H\mathbb{Z}$-local.

3. Let A be an abelian group. Show that there exists a connective spectrum $\mathbb{S}A$ such that
$$H\mathbb{Z}_*\mathbb{S}A = \begin{cases} A & \text{if } * = 0 \\ 0 & \text{otherwise} \end{cases} .$$
Show that if X is a spectrum there is a short exact sequence
$$0 \to \mathrm{Ext}(A, \pi_{*+1}X) \to \pi_* \operatorname{map}(\mathbb{S}A, X) \to \mathrm{Hom}(A, \pi_* X) \to 0\,,$$
where the second map sends $f : \mathbb{S}A \to \Sigma^n A$ to the induced map in π_0. Deduce from this that any two choices of $\mathbb{S}A$ are equivalent, but give an example of an A and an automorphism $f : \mathbb{S}A \to \mathbb{S}A$ that is not homotopic to the identity but induces the identity in integral homology.