

# Introduction to Stable Homotopy Theory

## Exercise Sheet 7

1. Let  $\kappa$  be a cardinal that is not regular. Show that a poset  $P$  is  $\kappa$ -filtered if and only if it is  $\kappa_+$ -filtered, where  $\kappa_+$  is the successor cardinal.
2. Let  $\kappa$  be a regular cardinal. Recall that a spectrum  $X$  is  $\kappa$ -compact if the functor  $\text{map}(X, -) : \text{Sp} \rightarrow \text{Sp}$  commutes with  $\kappa$ -filtered colimits. Show that a spectrum is  $\kappa$ -compact if and only if it is a retract of a  $\kappa$ -small spectrum.
3. Let  $X$  be a connective spectrum with trivial homology. Show that  $X = 0$ . Use the above result to show that a retract of a finite spectrum is finite (hint: go by induction on  $b - a$  where  $H_*E = 0$  if  $* < a$  or  $* > b$ ).