

Introduction to Stable Homotopy Theory

Exercise Sheet 5

1. Show that if X is an n -connected pointed space then $\pi_i \Sigma^\infty X = 0$ for $i \leq n$ (hint: use the Freudenthal suspension theorem). Deduce that the tensor product of two connective spectra is connective. (Hint: use the fact that if X is an $(n-1)$ -connected pointed space and Y is an $(m-1)$ -connected pointed space, then the smash product $X \wedge Y$ is $(n+m-1)$ -connected).

2. Let A, B be abelian groups. Show that there is an isomorphism of abelian groups

$$\pi_0(HA \otimes HB) \cong A \otimes_{\mathbb{Z}} B$$

(Hint use the Hurewicz isomorphism and the Künneth formula to compute $\pi_{n+m}(HA_n \wedge HB_m)$).

3. Let G be a discrete group. Show that its classifying space BG is equivalent to the nerve of the category with one object and G as a hom-set.