Introduction to Stable Homotopy Theory
Exercise Sheet 3

1. Let $\mathcal{C}$ and $\mathcal{D}$ be two categories. Show that there is an isomorphism of simplicial sets
$$N\,\mathrm{Fun}(\mathcal{C},\mathcal{D}) \cong \mathrm{Fun}(N\mathcal{C}, N\mathcal{D})$$
where $\mathrm{Fun}(\mathcal{C},\mathcal{D})$ is the category of functors from $\mathcal{C}$ to $\mathcal{D}$ and natural transformations.

2. Let $F : h\mathscr{S}_*{}^{\mathrm{op}} \to \mathrm{Set}$ be a functor such that for any collection $\{X_\alpha\}_{\alpha\in A}$ of pointed spaces the map
$$F\left(\bigvee_{\alpha\in A} X_\alpha\right) \to \prod_{\alpha\in A} F(X_\alpha)$$
is an isomorphism. Show that for $n \geq 1$ the set $F(S^n)$ has a canonical group structure, abelian if $n \geq 2$.

3. Suppose that E, F are two spectra and E^*, F^* are the corresponding cohomology theories. Show that if
$$\alpha^* : E^*(-) \to F^*(-)$$
is a natural transformation that commutes with the suspension isomorphisms, then there is $\alpha : E \to F$ map of spectra inducing α^*.