Introduction to Stable Homotopy Theory
Exercise Sheet 2

1. Let P be a partially ordered set. Show that there is a natural isomorphism

$$N(P) \cong \operatorname*{colim}_{S \subseteq P} N(S)$$

where S runs through all finite non-empty totally ordered subsets of P.

2. Let P be a partially ordered set. Let $\mathfrak{C}[P]$ be the simplicial category with objects the elements of P and

$$\operatorname{Map}_{\mathfrak{C}[P]}(p, q) = N\{S \subseteq P \mid S \text{ finite totally ordered, } \min S = p, \max S = q\}$$

where composition is given by the union of subsets. Show, using the previous exercise, that for every Kan-enriched category $\mathbf{C}$ there is a natural bijection

$$\operatorname{Hom}_{\mathrm{sSet}}(N(P), N^{\Delta}\mathbf{C}) \cong \operatorname{Hom}_{\mathrm{Cat}_{\Delta}}(\mathfrak{C}[P], \mathbf{C})$$

3. Let $\mathcal{C}$ be an ∞-category. Show that for every $x, y \in \operatorname{ob}\mathcal{C}$ there is a choice of composition map (as defined in class)

$$\circ : \operatorname{Map}_{\mathcal{C}}(x, x) \times \operatorname{Map}_{\mathcal{C}}(x, y) \to \operatorname{Map}_{\mathcal{C}}(x, y)$$

such that $f \circ \mathrm{id}_x = f$ for every $f \in \operatorname{Map}_{\mathcal{C}}(x, y)$.