Introduction to Stable Homotopy Theory
Exercise Sheet 1

1. Let Y be a topological space. Show that two n-simplices $\sigma, \tau \in (\operatorname{Sing} Y)([n])$ are homotopic relative to the boundary (in the sense explained in class) if and only if, when seen as maps $|\Delta^n| \to Y$ they are homotopic relative to the subspace $|\partial\Delta^n|$.

2. Let X, Y, Z be Kan complexes and let $f, f : X \to Y$ and $g, g' : Y \to Z$ be homotopic maps. Show that $g'f'$ and gf are homotopic.

3. Let X, Y be Kan complexes and $H : X \times \Delta^1 \to Y$ be a homotopy between two maps $f = H|_{X\times\{0\}}$ and $g = H|_{X\times\{1\}}$. Then for every $x \in X$ let γ be the path $H|_{\{x\}\times\Delta^1}$. Show that there's a commutative diagram

$$\begin{array}{ccc} \pi_n(X, x) & \xrightarrow{f_*} & \pi_n(Y, fx) \\ & \searrow^{g_*} & \downarrow{\scriptstyle \gamma_*} \\ & & \pi_n(Y, gx) \end{array} ,$$

   where γ_* is the isomorphism constructed in class.

   Deduce from the previous fact that homotopy equivalences induce isomorphisms between homotopy groups.