

Filtrated spectra part of integers w/ decreasing order

Def: A filtrated spectrum is a functor  $X: \mathbb{Z} \rightarrow \text{Sp}$

$$\dots \rightarrow X^{s+2} \rightarrow X^{s+1} \rightarrow X^s \rightarrow X^{s-1} \rightarrow \dots$$

$$X^\infty = \text{colim } X^s \text{ ("} \bigcup_s X^s \text{"}), \quad X^{-\infty} = \text{lim } X^s \text{ ("} \bigcap_s X^s \text{"})$$

$$g_i^s X = \text{cog}(X^{s+i} \rightarrow X^s) = X^i / X^{s+i} \text{ ("unrolled generator")}$$

IDEA: Use  $\pi_* g_i^s X$  to study  $\pi_* X^\infty, \pi_* X^{-\infty}$ .

Ex:  $E, X$  spectra. We have the Whitehead tower

$$t_{2n} E \rightarrow t_{2n-1} E \rightarrow t_{2n-2} E \rightarrow \dots \quad (t_{2n} E = \sum_{i=0}^n t_{2i} \Sigma^{-i} E)$$

$$t_{2n} E = 0, \quad t_{2n-1} E = E$$

$$g_i^s t_{2n} E = \Sigma^s H_{\pi_3} E$$

$$t_{2n} E \otimes X: \dots \rightarrow t_{2n-1} E \otimes X \rightarrow t_{2n-2} E \otimes X \rightarrow \dots$$

$$t_{2n-1} E \otimes X = E \otimes X, \quad g_i^s(t_{2n} E \otimes X) = \Sigma^s (H_{\pi_3} E \otimes X)$$

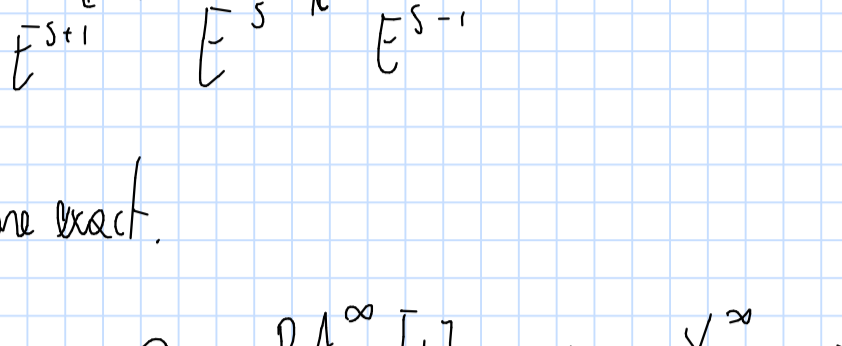
$$\pi_* g_i^s(t_{2n} E \otimes X) = H_{k-s}(X; \pi_3 E)$$

$$(X \text{ bounded below} \Rightarrow t_{2n} E \otimes X = 0)$$

$$\dots \rightarrow X^{s+2} \rightarrow X^{s+1} \rightarrow X^s \rightarrow X^{s-1} \rightarrow \dots$$

IDEA: Write the les of the cofiber seq.  $X^{s+1} \rightarrow X^s \rightarrow g_i^s X$ .

$$\dots \rightarrow \pi_* X^{s+2} \rightarrow \pi_* X^{s+1} \rightarrow \pi_* X^s \rightarrow \pi_* X^{s-1} \rightarrow \dots$$



$$A^s = \pi_* X^s$$

$$E^s = \pi_* g_i^s X$$

Def: An (unrolled) exact couple is a diagram

$$A = \text{lin } A^s \rightarrow A^{s+2} \xrightarrow{i} A^{s+1} \xrightarrow{j} A^s \xrightarrow{k} A^{s-1} \xrightarrow{l} \dots \rightarrow A^{-\infty} = \text{colim } A^s$$

$$RA^\infty = \text{lin } A^s \quad \begin{matrix} \downarrow k \\ E^{s+1} \end{matrix} \quad \begin{matrix} \downarrow j \\ E^s \end{matrix} \quad \begin{matrix} \downarrow i \\ E^{s-1} \end{matrix} \quad \begin{matrix} \downarrow l \\ E^{s-2} \end{matrix}$$

when the triangles are exact.

Major exact sequence:  $0 \rightarrow RA^\infty[1] \rightarrow \pi_* X^\infty \rightarrow A^\infty \rightarrow 0$

(les in the filter seq.  $X^\infty = \text{lin } X^s \rightarrow \prod X^s \xrightarrow{d_i} \prod X^s$ )

( $X^\infty = 0 \Rightarrow A^\infty = RA^\infty = 0$ )

the spectral sequence is conditionally convergent to  $X^\infty$

$$A^{-\infty} = \pi_* X^{-\infty} \quad (X^{-\infty} = 0 \text{ conditionally convergent to } X^\infty)$$

IDEA: Put filtrations on  $A^\infty, A^{-\infty}$

$$F^s A^{-\infty} = \text{Im}(A^s \rightarrow A^{-\infty}), \quad F^s A^\infty = \text{Ker}(A^\infty \rightarrow A^s) \text{ (obvious filtrations)}$$

Goal: Complete  $F^s A^{-\infty} / F^{s+1} A^{-\infty}, F^s A^\infty / F^{s+1} A^\infty$ .

$$\dots \rightarrow A^{s+2} \xrightarrow{i} A^{s+1} \xrightarrow{j} A^s \xrightarrow{k} A^{s-1} \xrightarrow{l} \dots \quad \begin{matrix} A^s = \pi_* X^s \\ E^s = \pi_* g_i^s X \end{matrix}$$

$$Z_2^s = \{x \in E^s \mid kx \in \text{Im}(A^{s+2} \rightarrow A^{s+1})\} \supseteq \text{Ker } k \quad \text{Im } j$$

$$B_2^s = \{x \in E^s \mid x = jy, y \in \text{Ker}(A^s \rightarrow A^{s+2})\} \subseteq \text{Ker } k \quad (k_j = 0)$$

$$Z_1^s \supseteq Z_2^s \supseteq Z_3^s \supseteq \dots \supseteq \text{Ker } k = \text{Im } j \supseteq \dots \supseteq B_3^s \supseteq B_2^s \supseteq B_1^s$$

$$E_2^s = Z_2^s / B_2^s \quad d_2: E_2^s \rightarrow E_2^{s+2} \quad \text{Ker } d_2 = Z_{21}^s / B_2^s, \quad \text{Im } d_2 = B_{21}^{s+1} / B_2^{s+1}$$

$$E_{21}^s = \text{Ker } d_2 / \text{Im } d_2$$

$$d_2(x) = jy \quad \text{where } i^2 y = kx.$$

Exercise: Verify this is well-defined w/ the properties I claim.

$$Z_\infty^s = \bigcap_{i \geq 0} Z_i^s \text{ (permanent cycles)} \quad B_\infty^s = \bigcup_{i \geq 0} B_i^s \text{ (boundaries)}$$

$$E_\infty^s := Z_\infty^s / B_\infty^s$$

Proposition: Suppose  $\lim_z Z_z^s = 0 \forall s$ , then  $j$  is SES

$$0 \rightarrow F^s A^{-\infty} / F^{s+1} A^{-\infty} \rightarrow E_\infty^s \xrightarrow{\cong} F^s A^\infty / F^{s+1} A^\infty \rightarrow 0$$

$$F^s A^{-\infty} = \text{Im}(A^s \rightarrow A^{-\infty}) \xrightarrow{\cong} E_\infty^s$$

$j\tilde{x} \in B_\infty^s \Leftrightarrow \tilde{x}$  can be dropped on that side in  $\text{Im } i: A^{s+1} \rightarrow A^s$ .

Ex: If  $A^\infty = 0 \Rightarrow E_\infty^s = F^s A^{-\infty} / F^{s+1} A^{-\infty}$ .

$$E_\infty^s \Rightarrow A^\infty \rightarrow \underline{E}_s: H_p(X; \pi_q E) \Rightarrow \pi_{p+q}(E \otimes X)$$

Adams spectral sequence

Throughout we'll fix a prime  $p$ .

Recall: A map  $f: X \rightarrow Y$  is trivial on homology if  $H\mathbb{F}_p \otimes f \sim 0$ .

Def: Let  $X, Y$  be spectra. The Adams filtration or  $\pi_* \text{map}(X, Y)$  is the filtration

$$F^s \pi_* \text{map}(X, Y) = \{f: X \rightarrow Y \mid f = f_s \circ \dots \circ f_1, f_i \text{ trivial on homology}\}$$

$$\text{Ex: } p \in \pi_0 S \text{ is in Adams filtration, } X = Z^0 \xrightarrow{d_1} Z^1 \rightarrow \dots \xrightarrow{d_s} Z^s = Y$$

Lemma:  $f: X \rightarrow Y$  is trivial on homology iff the composition  $X \rightarrow Y \rightarrow H\mathbb{F}_p \otimes Y$  is nullhomotopic.

Proof:  $X \rightarrow H\mathbb{F}_p \otimes X \xrightarrow{H\mathbb{F}_p \otimes f} H\mathbb{F}_p \otimes Y \xrightarrow{\text{log}} H\mathbb{F}_p \otimes H\mathbb{F}_p \otimes Y$

Def:  $\overline{H\mathbb{F}_p} = \text{fib}(S \rightarrow H\mathbb{F}_p) \Rightarrow f$  is trivial on homology  $\Leftrightarrow$  it factors through  $\overline{H\mathbb{F}_p} \otimes Y$

Remark:  $f$  is in Adams filtration iff it factors through  $\overline{H\mathbb{F}_p}^{\otimes s} \otimes Y \rightarrow Y$

$$X = Z^0 \xrightarrow{d_1} Z^1 \xrightarrow{d_2} Z^2 \xrightarrow{d_3} Z^3 \xrightarrow{d_4} Z^4 \xrightarrow{d_5} Z^5 \xrightarrow{d_6} Z^6 \xrightarrow{d_7} Z^7 \xrightarrow{d_8} Z^8 \xrightarrow{d_9} Z^9 \xrightarrow{d_{10}} Z^{10} \xrightarrow{d_{11}} Z^{11} \xrightarrow{d_{12}} Z^{12} \xrightarrow{d_{13}} Z^{13} \xrightarrow{d_{14}} Z^{14} \xrightarrow{d_{15}} Z^{15} \xrightarrow{d_{16}} Z^{16} \xrightarrow{d_{17}} Z^{17} \xrightarrow{d_{18}} Z^{18} \xrightarrow{d_{19}} Z^{19} \xrightarrow{d_{20}} Z^{20} \xrightarrow{d_{21}} Z^{21} \xrightarrow{d_{22}} Z^{22} \xrightarrow{d_{23}} Z^{23} \xrightarrow{d_{24}} Z^{24} \xrightarrow{d_{25}} Z^{25} \xrightarrow{d_{26}} Z^{26} \xrightarrow{d_{27}} Z^{27} \xrightarrow{d_{28}} Z^{28} \xrightarrow{d_{29}} Z^{29} \xrightarrow{d_{30}} Z^{30} \xrightarrow{d_{31}} Z^{31} \xrightarrow{d_{32}} Z^{32} \xrightarrow{d_{33}} Z^{33} \xrightarrow{d_{34}} Z^{34} \xrightarrow{d_{35}} Z^{35} \xrightarrow{d_{36}} Z^{36} \xrightarrow{d_{37}} Z^{37} \xrightarrow{d_{38}} Z^{38} \xrightarrow{d_{39}} Z^{39} \xrightarrow{d_{40}} Z^{40} \xrightarrow{d_{41}} Z^{41} \xrightarrow{d_{42}} Z^{42} \xrightarrow{d_{43}} Z^{43} \xrightarrow{d_{44}} Z^{44} \xrightarrow{d_{45}} Z^{45} \xrightarrow{d_{46}} Z^{46} \xrightarrow{d_{47}} Z^{47} \xrightarrow{d_{48}} Z^{48} \xrightarrow{d_{49}} Z^{49} \xrightarrow{d_{50}} Z^{50} \xrightarrow{d_{51}} Z^{51} \xrightarrow{d_{52}} Z^{52} \xrightarrow{d_{53}} Z^{53} \xrightarrow{d_{54}} Z^{54} \xrightarrow{d_{55}} Z^{55} \xrightarrow{d_{56}} Z^{56} \xrightarrow{d_{57}} Z^{57} \xrightarrow{d_{58}} Z^{58} \xrightarrow{d_{59}} Z^{59} \xrightarrow{d_{60}} Z^{60} \xrightarrow{d_{61}} Z^{61} \xrightarrow{d_{62}} Z^{62} \xrightarrow{d_{63}} Z^{63} \xrightarrow{d_{64}} Z^{64} \xrightarrow{d_{65}} Z^{65} \xrightarrow{d_{66}} Z^{66} \xrightarrow{d_{67}} Z^{67} \xrightarrow{d_{68}} Z^{68} \xrightarrow{d_{69}} Z^{69} \xrightarrow{d_{70}} Z^{70} \xrightarrow{d_{71}} Z^{71} \xrightarrow{d_{72}} Z^{72} \xrightarrow{d_{73}} Z^{73} \xrightarrow{d_{74}} Z^{74} \xrightarrow{d_{75}} Z^{75} \xrightarrow{d_{76}} Z^{76} \xrightarrow{d_{77}} Z^{77} \xrightarrow{d_{78}} Z^{78} \xrightarrow{d_{79}} Z^{79} \xrightarrow{d_{80}} Z^{80} \xrightarrow{d_{81}} Z^{81} \xrightarrow{d_{82}} Z^{82} \xrightarrow{d_{83}} Z^{83} \xrightarrow{d_{84}} Z^{84} \xrightarrow{d_{85}} Z^{85} \xrightarrow{d_{86}} Z^{86} \xrightarrow{d_{87}} Z^{87} \xrightarrow{d_{88}} Z^{88} \xrightarrow{d_{89}} Z^{89} \xrightarrow{d_{90}} Z^{90} \xrightarrow{d_{91}} Z^{91} \xrightarrow{d_{92}} Z^{92} \xrightarrow{d_{93}} Z^{93} \xrightarrow{d_{94}} Z^{94} \xrightarrow{d_{95}} Z^{95} \xrightarrow{d_{96}} Z^{96} \xrightarrow{d_{97}} Z^{97} \xrightarrow{d_{98}} Z^{98} \xrightarrow{d_{99}} Z^{99} \xrightarrow{d_{100}} Z^{100} \xrightarrow{d_{101}} Z^{101} \xrightarrow{d_{102}} Z^{102} \xrightarrow{d_{103}} Z^{103} \xrightarrow{d_{104}} Z^{104} \xrightarrow{d_{105}} Z^{105} \xrightarrow{d_{106}} Z^{106} \xrightarrow{d_{107}} Z^{107} \xrightarrow{d_{108}} Z^{108} \xrightarrow{d_{109}} Z^{109} \xrightarrow{d_{110}} Z^{110} \xrightarrow{d_{111}} Z^{111} \xrightarrow{d_{112}} Z^{112} \xrightarrow{d_{113}} Z^{113} \xrightarrow{d_{114}} Z^{114} \xrightarrow{d_{115}} Z^{115} \xrightarrow{d_{116}} Z^{116} \xrightarrow{d_{117}} Z^{117} \xrightarrow{d_{118}} Z^{118} \xrightarrow{d_{119}} Z^{119} \xrightarrow{d_{120}} Z^{120} \xrightarrow{d_{121}} Z^{121} \xrightarrow{d_{122}} Z^{122} \xrightarrow{d_{123}} Z^{123} \xrightarrow{d_{124}} Z^{124} \xrightarrow{d_{125}} Z^{125} \xrightarrow{d_{126}} Z^{126} \xrightarrow{d_{127}} Z^{127} \xrightarrow{d_{128}} Z^{128} \xrightarrow{d_{129}} Z^{129} \xrightarrow{d_{130}} Z^{130} \xrightarrow{d_{131}} Z^{131} \xrightarrow{d_{132}} Z^{132} \xrightarrow{d_{133}} Z^{133} \xrightarrow{d_{134}} Z^{134} \xrightarrow{d_{135}} Z^{135} \xrightarrow{d_{136}} Z^{136} \xrightarrow{d_{137}} Z^{137} \xrightarrow{d_{138}} Z^{138} \xrightarrow{d_{139}} Z^{139} \xrightarrow{d_{140}} Z^{140} \xrightarrow{d_{141}} Z^{141} \xrightarrow{d_{142}} Z^{142} \xrightarrow{d_{143}} Z^{143} \xrightarrow{d_{144}} Z^{144} \xrightarrow{d_{145}} Z^{145} \xrightarrow{d_{146}} Z^{146} \xrightarrow{d_{147}} Z^{147} \xrightarrow{d_{148}} Z^{148} \xrightarrow{d_{149}} Z^{149} \xrightarrow{d_{150}} Z^{150} \xrightarrow{d_{151}} Z^{151} \xrightarrow{d_{152}} Z^{152} \xrightarrow{d_{153}} Z^{153} \xrightarrow{d_{154}} Z^{154} \xrightarrow{d_{155}} Z^{155} \xrightarrow{d_{156}} Z^{156} \xrightarrow{d_{157}} Z^{157} \xrightarrow{d_{158}} Z^{158} \xrightarrow{d_{159}} Z^{159} \xrightarrow{d_{160}} Z^{160} \xrightarrow{d_{161}} Z^{161} \xrightarrow{d_{162}} Z^{162} \xrightarrow{d_{163}} Z^{163} \xrightarrow{d_{164}} Z^{164} \xrightarrow{d_{165}} Z^{165} \xrightarrow{d_{166}} Z^{166} \xrightarrow{d_{167}} Z^{167} \xrightarrow{d_{168}} Z^{168} \xrightarrow{d_{169}} Z^{169} \xrightarrow{d_{170}} Z^{170} \xrightarrow{d_{171}} Z^{171} \xrightarrow{d_{172}} Z^{172} \xrightarrow{d_{173}} Z^{173} \xrightarrow{d_{174}} Z^{174} \xrightarrow{d_{175}} Z^{175} \xrightarrow{d_{176}} Z^{176} \xrightarrow{d_{177}} Z^{177} \xrightarrow{d_{178}} Z^{178} \xrightarrow{d_{179}} Z^{179} \xrightarrow{d_{180}} Z^{180} \xrightarrow{d_{181}} Z^{181} \xrightarrow{d_{182}} Z^{182} \xrightarrow{d_{183}} Z^{183} \xrightarrow{d_{184}} Z^{184} \xrightarrow{d_{185}} Z^{185} \xrightarrow{d_{186}} Z^{186} \xrightarrow{d_{187}} Z^{187} \xrightarrow{d_{188}} Z^{188} \xrightarrow{d_{189}} Z^{189} \xrightarrow{d_{190}} Z^{190} \xrightarrow{d_{191}} Z^{191} \xrightarrow{d_{192}} Z^{192} \xrightarrow{d_{193}} Z^{193} \xrightarrow{d_{194}} Z^{194} \xrightarrow{d_{195}} Z^{195} \xrightarrow{d_{196}} Z^{196} \xrightarrow{d_{197}} Z^{197} \xrightarrow{d_{198}} Z^{198} \xrightarrow{d_{199}} Z^{199} \xrightarrow{d_{200}} Z^{200} \xrightarrow{d_{201}} Z^{201} \xrightarrow{d_{202}} Z^{202} \xrightarrow{d_{203}} Z^{203} \xrightarrow{d_{204}} Z^{204} \xrightarrow{d_{205}} Z^{205} \xrightarrow{d_{206}} Z^{206} \xrightarrow{d_{207}} Z^{207} \xrightarrow{d_{208}} Z^{208} \xrightarrow{d_{209}} Z^{209} \xrightarrow{d_{210}} Z^{210} \xrightarrow{d_{211}} Z^{211} \xrightarrow{d_{212}} Z^{212} \xrightarrow{d_{213}} Z^{213} \xrightarrow{d_{214}} Z^{214} \xrightarrow{d_{215}} Z^{215} \xrightarrow{d_{216}} Z^{216} \xrightarrow{d_{217}} Z^{217} \xrightarrow{d_{218}} Z^{218} \xrightarrow{d_{219}} Z^{219} \xrightarrow{d_{220}} Z^{220} \xrightarrow{d_{221}} Z^{221} \xrightarrow{d_{222}} Z^{222} \xrightarrow{d_{223}} Z^{223} \xrightarrow{d_{224}} Z^{224} \xrightarrow{d_{225}} Z^{225} \xrightarrow{d_{226}} Z^{226} \xrightarrow{d_{227}} Z^{227} \xrightarrow{d_{228}} Z^{228} \xrightarrow{d_{229}} Z^{229} \xrightarrow{d_{230}} Z^{230} \xrightarrow{d_{231}} Z^{231} \xrightarrow{d_{232}} Z^{232} \xrightarrow{d_{233}} Z^{233} \xrightarrow{d_{234}} Z^{234} \xrightarrow{d_{235}} Z^{235} \xrightarrow{d_{236}} Z^{236} \xrightarrow{d_{237}} Z^{237} \xrightarrow{d_{238}} Z^{238} \xrightarrow{d_{239}} Z^{239} \xrightarrow{d_{240}} Z^{240} \xrightarrow{d_{241}} Z^{241} \xrightarrow{d_{242}} Z^{242} \xrightarrow{d_{243}} Z^{243} \xrightarrow{d_{244}} Z^{244} \xrightarrow{d_{245}} Z^{245} \xrightarrow{d_{246}} Z^{246} \xrightarrow{d_{247}} Z^{247} \xrightarrow{d_{248}} Z^{248} \xrightarrow{d_{249}} Z^{249} \xrightarrow{d_{250}} Z^{250} \xrightarrow{d_{251}} Z^{251} \xrightarrow{d_{252}} Z^{252} \xrightarrow{d_{253}} Z^{253} \xrightarrow{d_{254}} Z^{254} \xrightarrow{d_{255}} Z^{255} \xrightarrow{d_{256}} Z^{256} \xrightarrow{d_{257}} Z^{257} \xrightarrow{d_{258}} Z^{258} \xrightarrow{d_{259}} Z^{259} \xrightarrow{d_{260}} Z^{260} \xrightarrow{d_{261}} Z^{261} \xrightarrow{d_{262}} Z^{262} \xrightarrow{d_{263}} Z^{263} \xrightarrow{d_{264}} Z^{264} \xrightarrow{d_{265}} Z^{265} \xrightarrow{d_{266}} Z^{266} \xrightarrow{d_{267}} Z^{267} \xrightarrow{d_{268}} Z^{268} \xrightarrow{d_{269}} Z^{269} \xrightarrow{d_{270}} Z^{270} \xrightarrow{d_{271}} Z^{271} \xrightarrow{d_{272}} Z^{272} \xrightarrow{d_{273}} Z^{273} \xrightarrow{d_{274}} Z^{274} \xrightarrow{d_{275}} Z^{275} \xrightarrow{d_{276}} Z^{276} \xrightarrow{d_{277}} Z^{277} \xrightarrow{d_{278}} Z^{278} \xrightarrow{d_{279}} Z^{279} \xrightarrow{d_{280}} Z^{280} \xrightarrow{d_{281}} Z^{281} \xrightarrow{d_{282}} Z^{282} \xrightarrow{d_{283}} Z^{283} \xrightarrow{d_{284}} Z^{284} \xrightarrow{d_{285}} Z^{285} \xrightarrow{d_{286}} Z^{286} \xrightarrow{d_{287}} Z^{287} \xrightarrow{d_{288}} Z^{288} \xrightarrow{d_{289}} Z^{289} \xrightarrow{d_{290}} Z^{290} \xrightarrow{d_{291}} Z^{291} \xrightarrow{d_{292}} Z^{292} \xrightarrow{d_{293}} Z^{293} \xrightarrow{d_{294}} Z^{294} \xrightarrow{d_{295}} Z^{295} \xrightarrow{d_{296}} Z^{296} \xrightarrow{d_{297}} Z^{297} \xrightarrow{d_{298}} Z^{298} \xrightarrow{d_{299}} Z^{299} \xrightarrow{d_{300}} Z^{300} \xrightarrow{d_{301}} Z^{301} \xrightarrow{d_{302}} Z^{302} \xrightarrow{d_{303}} Z^{303} \xrightarrow{d_{304}} Z^{304} \xrightarrow{d_{305}} Z^{305} \xrightarrow{d_{306}} Z^{306} \xrightarrow{d_{307}} Z^{307} \xrightarrow{d_{308}} Z^{308} \xrightarrow{d_{309}} Z^{309} \xrightarrow{d_{310}} Z^{310} \xrightarrow{d_{311}} Z^{311} \xrightarrow{d_{312}} Z^{312} \xrightarrow{d_{313}} Z^{313} \xrightarrow{d_{314}} Z^{314} \xrightarrow{d_{315}} Z^{315} \xrightarrow{d_{316}} Z^{316} \xrightarrow{d_{317}} Z^{317} \xrightarrow{d_{318}} Z^{318} \xrightarrow{d_{319}} Z^{319} \xrightarrow{d_{320}} Z^{320} \xrightarrow{d_{321}} Z^{321} \xrightarrow{d_{322}} Z^{322} \xrightarrow{d_{323}} Z^{323} \xrightarrow{d_{324}} Z^{324} \xrightarrow{d_{325}} Z^{325} \xrightarrow{d_{326}} Z^{326} \xrightarrow{d_{327}} Z^{327} \xrightarrow{d_{328}} Z^{328} \xrightarrow{d_{329}} Z^{329} \xrightarrow{d_{330}} Z^{330} \xrightarrow{d_{331}} Z^{331} \xrightarrow{d_{332}} Z^{332} \xrightarrow{d_{333}} Z^{333} \xrightarrow{d_{334}} Z^{334} \xrightarrow{d_{335}} Z^{335} \xrightarrow{d_{336}} Z^{336} \xrightarrow{d_{337}} Z^{337} \xrightarrow{d_{338}} Z^{338} \xrightarrow{d_{339}} Z^{339} \xrightarrow{d_{340}} Z^{340} \xrightarrow{d_{341}} Z^{341} \xrightarrow{d_{342}} Z^{342} \xrightarrow{d_{343}} Z^{343} \xrightarrow{d_{344}} Z^{344} \xrightarrow{d_{345}} Z^{345} \xrightarrow{d_{346}} Z^{346} \xrightarrow{d_{347}} Z^{347} \xrightarrow{d_{348}} Z^{348} \xrightarrow{d_{349}} Z^{349} \xrightarrow{d_{350}} Z^{350} \xrightarrow{d_{351}} Z^{351} \xrightarrow{d_{352}} Z^{352} \xrightarrow{d_{353}} Z^{353} \xrightarrow{d_{354}} Z^{354} \xrightarrow{d_{355}} Z^{355} \xrightarrow{d_{356}} Z^{356} \xrightarrow{d_{357}} Z^{357} \xrightarrow{d_{358}} Z^{358} \xrightarrow{d_{359}} Z^{359} \xrightarrow{d_{360}} Z^{360} \xrightarrow{d_{361}} Z^{361} \xrightarrow{d_{362}} Z^{362} \xrightarrow{d_{363}} Z^{363} \xrightarrow{d_{364}} Z^{364} \xrightarrow{d_{365}} Z^{365} \xrightarrow{d_{366}} Z^{366} \xrightarrow{d_{367}} Z^{367} \xrightarrow{d_{368}} Z^{368} \xrightarrow{d_{369}} Z^{369} \xrightarrow{d_{370}} Z^{370} \xrightarrow{d_{371}} Z^{371} \xrightarrow{d_{372}} Z^{372} \xrightarrow{d_{373}} Z^{373} \xrightarrow{d_{374}} Z^{374} \xrightarrow{d_{375}} Z^{375} \xrightarrow{d_{376}} Z^{376} \xrightarrow{d_{377}} Z^{377} \xrightarrow{d_{378}} Z^{378} \xrightarrow{d_{379}} Z^{379} \xrightarrow{d_{380}} Z^{380} \xrightarrow{d_{381}} Z^{381} \xrightarrow{d_{382}} Z^{382} \xrightarrow{d_{383}} Z^{383} \xrightarrow{d_{384}} Z^{384} \xrightarrow{d_{385}} Z^{385} \xrightarrow{d_{386}} Z^{386} \xrightarrow{d_{387}} Z^{387} \xrightarrow{d_{388}} Z^{388} \xrightarrow{d_{389}} Z^{389} \xrightarrow{d_{390}} Z^{390} \xrightarrow{d_{391}} Z^{391} \xrightarrow{d_{392}} Z^{392} \xrightarrow{d_{393}} Z^{393} \xrightarrow{d_{394}} Z^{394} \xrightarrow{d_{395}} Z^{395} \xrightarrow{d_{396}} Z^{396} \xrightarrow{d_{397}} Z^{397} \xrightarrow{d_{398}} Z^{398} \xrightarrow{d_{399}} Z^{399} \xrightarrow{d_{400}} Z^{400} \xrightarrow{d_{401}} Z^{401} \xrightarrow{d_{402}} Z^{402} \xrightarrow{d_{403}} Z^{403} \xrightarrow{d_{404}} Z^{404} \xrightarrow{d_{405}} Z^{405} \xrightarrow{d_{406}} Z^{406} \xrightarrow{d_{407}} Z^{407} \xrightarrow{d_{408}} Z^{408} \xrightarrow{d_{409}} Z^{409} \xrightarrow{d_{410}} Z^{410} \xrightarrow{d_{411}} Z^{411} \xrightarrow{d_{412}} Z^{412} \xrightarrow{d_{413}} Z^{413} \xrightarrow{d_{414}} Z^{414} \xrightarrow{d_{415}} Z^{415} \xrightarrow{d_{416}} Z^{416} \xrightarrow{d_{417}} Z^{417} \xrightarrow{d_{418}} Z^{418} \xrightarrow{d_{419}} Z^{419} \xrightarrow{d_{420}} Z^{420} \xrightarrow{d_{421}} Z^{421} \xrightarrow{d_{422}} Z^{422} \xrightarrow{d_{423}} Z^{423} \xrightarrow{d_{424}} Z^{424} \xrightarrow{d_{425}} Z^{425} \xrightarrow{d_{426}} Z^{426} \xrightarrow{d_{427}} Z^{427} \xrightarrow{d_{428}} Z^{428} \xrightarrow{d_{429}} Z^{429} \xrightarrow{d_{430}} Z^{430} \xrightarrow{d_{431}} Z^{431} \xrightarrow{d_{432}} Z^{432} \xrightarrow{d_{433}} Z^{433} \xrightarrow{d_{434}} Z^{434} \xrightarrow{d_{435}} Z^{435} \xrightarrow{d_{436}} Z^{436} \xrightarrow{d_{437}} Z^{437} \xrightarrow{d_{438}} Z^{438} \xrightarrow{d_{439}} Z^{439} \xrightarrow{d_{440}} Z^{440} \xrightarrow{d_{441}} Z^{441} \xrightarrow{d_{442}} Z^{442} \xrightarrow{d_{443}} Z^{443} \xrightarrow{d_{444}} Z^{444} \xrightarrow{d_{445}} Z^{445} \xrightarrow{d_{446}} Z^{446} \xrightarrow{d_{447}} Z^{447} \xrightarrow{d_{448}} Z^{448} \xrightarrow{d_{449}} Z^{449} \xrightarrow{d_{450}} Z^{450} \xrightarrow{d_{451}} Z^{451} \xrightarrow{d_{452}} Z^{452} \xrightarrow{d_{453}} Z^{453} \xrightarrow{d_{454}} Z^{454} \xrightarrow{d_{455}} Z^{455} \xrightarrow{d_{456}} Z^{456} \xrightarrow{d_{457}} Z^{457} \xrightarrow{d_{458}} Z^{458} \xrightarrow{d_{459}} Z^{459} \xrightarrow{d_{460}} Z^{460} \xrightarrow{d_{461}} Z^{461} \xrightarrow{d_{462}} Z^{462} \xrightarrow{d_{463}} Z^{463} \xrightarrow{d_{464}} Z^{464} \xrightarrow{d_{465}} Z^{465} \xrightarrow{d_{466}} Z^{466} \xrightarrow{d_{467}} Z^{467} \xrightarrow{d_{468}} Z^{468} \xrightarrow{d_{469}} Z^{469} \xrightarrow{d_{470}} Z^{470} \xrightarrow{d_{471}} Z^{471} \xrightarrow{d_{472}} Z^{472} \xrightarrow{d_{473}} Z^{473} \xrightarrow{d_{474}} Z^{474} \xrightarrow{d_{475}} Z^{475} \xrightarrow{d_{476}} Z^{476} \xrightarrow{d_{4$$