Appendices

A Collection of formulas

Dirac algebra in 4 Dimensions

Traces with even number of γ-matrices

$$\mathrm{Tr}\{\mathbf{1}\} = 4 \tag{A.1}$$

$$\mathrm{Tr}\{\gamma_\mu\gamma_\nu\} = 4g_{\mu\nu} \tag{A.2}$$

$$\mathrm{Tr}\{\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\beta\} = 4[g_{\mu\nu}g_{\alpha\beta} + g_{\mu\beta}g_{\nu\alpha} - g_{\mu\alpha}g_{\nu\beta}] \tag{A.3}$$

Traces with odd number of γ-matrices

$$\mathrm{Tr}\{\gamma_{\mu_1}\ldots\gamma_{\mu_{2k+1}}\} = 0\,, \qquad k = 0,1,2,\ldots \tag{A.4}$$

Traces including a γ_5-matrix

$$\mathrm{Tr}\{\gamma_5\} = 0 \tag{A.5}$$

$$\mathrm{Tr}\{\gamma_\mu\gamma_\nu\gamma_5\} = 0 \tag{A.6}$$

$$\mathrm{Tr}\{\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\beta\gamma_5\} = 4i\epsilon_{\mu\nu\alpha\beta} \tag{A.7}$$

$$\mathrm{Tr}\{\gamma_{\mu_1}\ldots\gamma_{\mu_{2k+1}}\gamma_5\} = 0\,, \qquad k = 0,1,2,\ldots \tag{A.8}$$

Useful identities for products of γ-matrices

$$\gamma_\mu\gamma^\mu = 4 \tag{A.9}$$

$$\gamma_\mu\gamma_\alpha\gamma^\mu = -2\gamma_\alpha \tag{A.10}$$

$$\gamma_\mu\gamma_\alpha\gamma_\beta\gamma^\mu = 4g_{\alpha\beta} \tag{A.11}$$

$$\gamma_\mu\gamma_\alpha\gamma_\beta\gamma_\rho\gamma^\mu = -2\gamma_\rho\gamma_\beta\gamma_\alpha \tag{A.12}$$

$$\gamma_\mu\gamma_\alpha\gamma_\nu = g_{\alpha\mu}\gamma_\nu + g_{\alpha\nu}\gamma_\mu - g_{\mu\nu}\gamma_\alpha + i\epsilon_{\mu\alpha\nu\beta}\gamma_5\gamma_\beta \tag{A.13}$$

Useful identities for products of ϵ-tensors

$$\epsilon_{\alpha\beta\mu\nu}\epsilon^{\alpha\beta\mu\nu} = -24 \tag{A.14}$$

$$\epsilon_{\alpha\beta\mu\nu}\epsilon^{\rho\beta\mu\nu} = -6g^\rho_\alpha \tag{A.15}$$

$$\epsilon_{\alpha\beta\mu\nu}\epsilon^{\rho\sigma\mu\nu} = -2[g^\rho_\alpha g^\sigma_\beta - g^\sigma_\alpha g^\rho_\beta] \tag{A.16}$$

$$\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}\epsilon^{\beta_1\beta_2\beta_3\beta_4} = -\det\left(g^{\beta_k}_{\alpha_i}\right) \tag{A.17}$$

$$\frac{1}{2}\epsilon_{\alpha\beta\mu\nu}\sigma^{\mu\nu} = i\sigma_{\alpha\beta}\gamma_5 \tag{A.18}$$

!!! We use definitions from Bjorken and Drell:

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3\,, \qquad \epsilon_{0123} = +1 \tag{A.19}$$

Be careful, some other (equally famous) books use different definitions:

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3\,, \qquad \epsilon^{0123} = -\epsilon_{0123} = +1 \qquad \text{Itzykson, Zuber} \tag{A.20}$$

$$\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3 = -i\gamma^0\gamma^1\gamma^2\gamma^3\,, \qquad \epsilon^{0123} = -\epsilon_{0123} = +1 \qquad \text{Okun} \tag{A.21}$$

This ambiguity is a standard source of sign errors!

Identities involving Dirac spinors

$$\begin{aligned} \bar{u}^\lambda(p)\, u^{\lambda'}(p) &= 2m\delta_{\lambda\lambda'} \\ \bar{v}^\lambda(p)\, v^{\lambda'}(p) &= -2m\delta_{\lambda\lambda'} \\ \bar{u}^\lambda(p)\, v^{\lambda'}(p) &= \bar{v}^\lambda(p)\, u^{\lambda'}(p) \;=\; 0 \end{aligned} \tag{A.22}$$

$$\bar{u}^\lambda(p)\gamma_\mu u^{\lambda'}(p) = \bar{v}^\lambda(p)\gamma_\mu v^{\lambda'}(p) \;=\; 2p_\mu\delta_{\lambda\lambda'} \tag{A.23}$$

$$\sum_{\lambda=\pm 1/2}\left[u^\lambda_\alpha(p)\bar{u}^\lambda_\beta(p) - v^\lambda_\alpha(p)\bar{v}^\lambda_\beta(p)\right] = 2m\delta_{\alpha\beta} = 2m(\mathbb{I})_{\alpha\beta} \tag{A.24}$$

$$\begin{aligned} \sum_{\lambda=\pm 1/2} u^\lambda_\alpha(p)\bar{u}^\lambda_\beta(p) &= (\not{p} + m)_{\alpha\beta} \\ \sum_{\lambda=\pm 1/2} v^\lambda_\alpha(p)\bar{v}^\lambda_\beta(p) &= (\not{p} - m)_{\alpha\beta} \end{aligned} \tag{A.25}$$

Hermitian and Charge conjugation

$$\gamma^0\gamma^\dagger_\nu\gamma^0 = \gamma_\nu \tag{A.26}$$

$$C = i\gamma^2\gamma^0\,, \qquad C^{-1}\gamma_\mu C = -\gamma_\mu^T\,, \qquad C = -C^{-1} = -C^\dagger = -C^T \tag{A.27}$$

Integration in the 4 dimensional Euclidean space

Definitions:

$$k_o \to ik_4 \tag{A.28}$$

$$d^4k \;=\; dk_o d^3\vec{k} = id^4k_E \tag{A.29}$$

$$k^2 \;=\; k_0^2 - \vec{k}^2 = -(k_1^2 + k_2^2 + k_3^2 + k_4^2) = -k_E^2 \tag{A.30}$$

Integration:

$$\int d^D k_E f(k_E^2) = \int d\Omega_D \int_0^\infty dk_E k_E^{D-1} f(k_E^2) \tag{A.31}$$

$$= \frac{\pi^{\frac{D}{2}}}{\Gamma\left(\frac{D}{2}\right)} \int_0^\infty dk_E^2 \left(k_E^2\right)^{\frac{D}{2}-1} f(k_E^2) \tag{A.32}$$

Dimensional Regularization ($D = 4 - 2\epsilon$)

Definitions:

$$\int d^4 k \to \int d^D k \tag{A.33}$$

$$e_0 \to e_0 \mu^{2-\frac{D}{2}} \tag{A.34}$$

Dirac algebra in D Dimensions

Defining the ϵ-tensor and γ_5 in D dimensions involves subtleties that would require a detailed explanation; we will leave out the corresponding formulas.

$$\gamma_\mu \gamma^\mu = D \tag{A.35}$$

$$\gamma_\mu \gamma_\alpha \gamma^\mu = (2 - D)\gamma_\alpha \tag{A.36}$$

$$\gamma_\mu \gamma_\alpha \gamma_\beta \gamma^\mu = 4 g_{\alpha\beta} + (D - 4)\gamma_\beta \gamma_\alpha \tag{A.37}$$

$$\gamma_\mu \gamma_\alpha \gamma_\beta \gamma_\rho \gamma^\mu = -2\gamma_\rho \gamma_\beta \gamma_\alpha + (4 - D)\gamma_\alpha \gamma_\beta \gamma_\rho \tag{A.38}$$

Feynman parameter integrals for products of propagators:

$$\frac{1}{A \cdot B} = \int_0^1 dx \frac{1}{[xA + (1-x)B]^2} \tag{A.39}$$

$$\frac{\Gamma(a)\Gamma(b)}{A^a \cdot B^b} = \int_0^1 dx\, dy\, x^{a-1} y^{b-1}\, \delta(1 - x - y) \frac{\Gamma(a+b)}{[xA + yB]^{a+b}}$$

$$= \int_0^1 dx\, x^{a-1} (1-x)^{b-1} \frac{\Gamma(a+b)}{[xA + (1-x)B]^{a+b}} \tag{A.40}$$

This representation can be generalized to an arbitrary number of the denominators, e.g.,

$$\frac{\Gamma(a)\Gamma(b)\Gamma(c)}{A^a \cdot B^b \cdot C^c} = \int_0^1 dx\, dy\, dz\, x^{a-1} y^{b-1} z^{c-1}\, \delta(1 - x - y - z) \frac{\Gamma(a+b+c)}{[xA + yB + zC]^{a+b+c}} \tag{A.41}$$

Loop integrals in D Dimensions

$$\int d^D k \, \frac{\Gamma(a)}{[-k^2 - A - i\epsilon]^a} = i\pi^{\frac{D}{2}} \frac{\Gamma\left(a - \frac{D}{2}\right)}{[-A - i\epsilon]^{a - \frac{D}{2}}} \tag{A.42}$$

$$\int d^D k \, \frac{\Gamma(a)}{[-k^2 - A - i\epsilon]^a} \, k_\mu k_\nu = i\pi^{\frac{D}{2}} \left(-\frac{g_{\mu\nu}}{2}\right) \frac{\Gamma\left(a - 1 - \frac{D}{2}\right)}{[-A - i\epsilon]^{a - 1 - \frac{D}{2}}} \tag{A.43}$$

$$\int d^D x \, \frac{\Gamma(\alpha)}{(-x^2 - a^2 + i\epsilon)^\alpha} = -i\pi^{D/2} \frac{\Gamma(\alpha - D/2)}{[-a^2 + i\epsilon]^{\alpha - D/2}}$$

$$\int d^D x \, \frac{\Gamma(\alpha)}{(-x^2 - a^2 + i\epsilon)^\alpha} x_\mu x_\nu = -i\pi^{D/2} \left(-\frac{g_{\mu\nu}}{2}\right) \frac{\Gamma(\alpha - D/2 - 1)}{[-a^2 + i\epsilon]^{\alpha - D/2 - 1}} \tag{A.44}$$

Fourier integrals in D Dimensions

$$\int d^D x \, e^{iqx} \frac{\Gamma(\alpha)}{[-x^2 + i\epsilon]^\alpha} = -i\pi^{D/2} 2^{D - 2\alpha} \frac{\Gamma(D/2 - \alpha)}{[-q^2 - i\epsilon]^{D/2 - \alpha}}$$

$$\int d^D q \, e^{-iqx} \frac{\Gamma(\alpha)}{[-q^2 - i\epsilon]^\alpha} = +i\pi^{D/2} 2^{D - 2\alpha} \frac{\Gamma(D/2 - \alpha)}{[-x^2 + i\epsilon]^{D/2 - \alpha}} \tag{A.45}$$