Universität Regensburg, Institut für Theoretische Physik WS23
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Exercises on Quantum Chromodynamics
problem sheet 12

Worksheet : Quark propagator in Background field and Fock-Schwinger gauge

The Quarkpropagator in background field has been introduced in the lecture as:

$$S(x) = \frac{i\not{x}}{2\pi^2 x^4} [x, 0]_{classic} + \mathcal{O}\left(\frac{1}{x^2}\right) \tag{1}$$

Problem 1

Determine the corrections of order $\mathcal{O}\left(\frac{1}{x^2}\right)$ for the coupling of one gluon. The result is:

$$\begin{aligned}\tilde{S}(x) = &-\frac{g}{2\pi^2}\frac{x^\mu \not{x}}{x^4}\int_0^1 du A_\mu(ux) \\ &- \frac{g}{8\pi^2}\frac{1}{x^2}\int_0^1 du \left[\partial_\alpha A_\mu(ux)\left(\bar{u}\not{x}\gamma^\mu\gamma^\alpha - u\gamma^\alpha\gamma^\mu\not{x}\right) + \bar{u}ux^\mu\not{x}\partial^2 A_\mu(ux)\right].\end{aligned} \tag{2}$$

(see Eq. (1.4) in Balitsky, Braun, Nuclear Physics B311 (1988/89), 541)

Problem 2

As we have seen in the last exercise sheet, in Fock-Schwinger gauge,

$$x_\mu A^\mu(x) = 0, \qquad A^\mu(x = 0) = 0, \tag{3}$$

the gauge field A^μ can be represented in the following form

$$A^\mu(x) = -\int_0^1 d\alpha \alpha x^\nu G_{\mu\nu}(\alpha x). \tag{4}$$

Show with the help of this representation, that the whole quark-propagator in Fock-Schwinger gauge can be expressed as follows:

$$\begin{aligned}S(x) = &\frac{i}{2\pi^2}\frac{\not{x}}{x^4} \\ &- \frac{ig}{16\pi^2}\frac{1}{x^2}\int_0^1 du \left[\bar{u}\not{x}\sigma^{\mu\nu}G_{\mu\nu}(ux) + u\sigma^{\mu\nu}G_{\mu\nu}(ux)\not{x} - 2i\bar{u}u\not{x}D^\mu G_{\mu\nu}x^\nu\right].\end{aligned} \tag{5}$$

(see Eq. (A.16) in Balitsky, Braun, Nuclear Physics B311 (1988/89), 541)

Comment on Problem 2:

The result you should obtain is valid up to order of $\sim G^2$ terms, which appear from this gauge, but also from diagrams with more than one gauge field. This means, any terms with multiple (more than one) gauge fields of A you can simply neglect.

Hint

It is most likely not a good idea to brute force insert the relation (4) into (2).
Instead it can be helpful to rewrite the gamma tensor into symmetry components, i.e. commutator and anticommutator.
For the remaining terms it might further be helpful to consider

$$\left(\frac{\partial}{\partial x^\nu}\right)^2 x^\mu A_\mu(ux) = \ ?$$

and to reduce a two fold integral to a one dimenensional one, i.e. to find the weighting w such that

$$\int_0^1 du \int_0^1 da \ u \ f(ua) = \int_0^1 du w(u) f(u). \tag{6}$$