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Exercises on Quantum Chromodynamics problem sheet 11

Worksheet : Fock-Schwinger Gauge and Wilson lines.

Problem 1

Prove that in Fock-Schwinger gauge

$$x_\mu A^\mu = 0 \qquad\qquad A^\mu(x=0) = 0 \tag{1}$$

the gauge field A^μ can be represented in the following form

$$A^\mu(x) = -\int_0^1 d\alpha\alpha x^\nu G_{\mu\nu}(\alpha x). \tag{2}$$

Hint: Consider the derivative $\partial_\mu\left(x^\nu A_\nu(\alpha x)\right)$.

Problem 2

Show that the P-ordered exponent along the straight line $(sx, 0)$

$$[sx, 0] \equiv P\exp\left\{ig\int_0^s d\tau x^\mu A_\mu(x\tau)\right\} \tag{3}$$

is a solution of the equation

$$\left(\frac{d}{ds} - ig\left(x^\mu A_\mu(xs)\right)\right)[xs, 0] = \left(x^\mu D_\mu\right)[xs, 0] = 0, \tag{4}$$

with the initial condition $[0, 0] = I$. Here $D_\mu = \partial_\mu - igA^a_\mu t^a$. Prove that under gauge transformations the P-ordered exponent transforms as

$$[x, 0; A'] = V(x)\,[x, 0; A]\,V^\dagger(0), \qquad A' = VAV^\dagger + \frac{i}{g}V\partial V^\dagger \tag{5}$$

Hint: Try to use the differential equation.

Problem 3
With the definition from Problem 2, try to prove that

$$\frac{\partial}{\partial x^\mu}\left[x,0\right] = ig\left(A_\mu(x)\left[x,0\right] + \int_0^1 d\tau\tau\left[x,\tau x\right]x^\nu F_{\mu,\nu}\left[\tau x,0\right]\right). \tag{6}$$

Hint: Define the function $F_\mu(s) = \frac{\partial}{\partial x^\mu}\left[sx,0\right]$. Try to obtain a differential equation for this function and solve it.