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Exercises on Quantum Chromodynamics
problem sheet 8

Worksheet : Quark condensate to the $\mathcal{T}$-product of two electromagnetic currents.

The total cross section of e^+e^- annihilation is related to the imaginary part of the amplitude

$$T_{\mu\nu} = i \int dx^4 e^{iqx} \langle 0|\mathcal{T}\left\{j_\mu(x) j_\nu(0)\right\}|0\rangle \tag{1}$$

where $j_\mu(0) = \overline{\psi}(x)\gamma_\mu\psi(x)$ is the electromagnetic current. (For simplicity, we consider the case with only one quark flavour and ignore electric charges). Following the work of K. Wilson, the $\mathcal{T}$-product of the currents can be expanded at $x_\mu \to 0$ in the series of contributions of renormalized local operators of increasing dimension

$$\langle 0|\mathcal{T}\left\{j_\mu(x) j_\nu(0)\right\}|0\rangle = \sum C^{(n)}_{\mu\nu} \langle 0|O^n(0)|0\rangle \tag{2}$$

where on the r.h.s. we have to retain contributions of all operators that may have a nonzero vacuum expectation value. In this exercise we consider operators of dimension three, built of a quark and an antiquark without derivatives.

Problem 1

Which of the operators $\overline{\psi}\Gamma\psi$, where $\Gamma = I_4, \gamma_5, \gamma_\mu, \sigma_{\mu\nu}$ may have nonzero vacuum expectation value (VEV) $\langle 0|\overline{\psi}\Gamma\psi|0\rangle \neq 0$? How could one write the general expression for the VEV of the product of the quark and antiquark field with open spinor i, k and color a, b indices

$$\langle 0|\psi^a_i \overline{\psi}^b_k|0\rangle = ? \tag{3}$$

Hint: Fierz Identity. Now also consider

$$\langle 0|D_\mu\psi^a_i \overline{\psi}^b_k|0\rangle. \tag{4}$$

You can use that the field opterators satisfy the equations of motion.

Problem 2

The coefficient function for the contribution of the quark-antiquark operators is given (to leading order) by the Feynman diagrams shown below, where the "legs" correspond to classical fields that for the VEV, and the propagators involve "quantum fields".

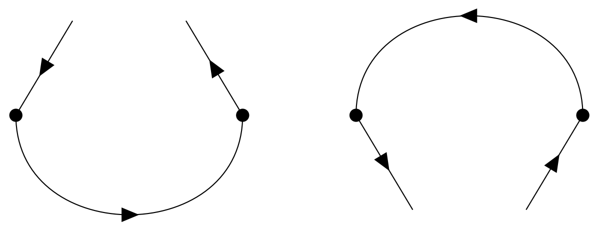

Figure 1: leading order Feynman diagrams related to $T_{\mu\nu}$

1. Show that for massless quarks these diagrams vanish
2. Calculate these diagrams assuming a small quark mass and neglecting the m^2 term in the denominator of quark propagator:

$$i\frac{\not{p} - m}{p^2 - m^2 + i\varepsilon} \to i\frac{\not{p} - m}{p^2 + i\varepsilon}$$

The corresponding coordinate space expression reads

$$\mathcal{T}\left\{\psi(x), \overline{\psi}(0)\right\} = \frac{i\Gamma\left(d/2\right)}{2\pi^{d/2}} \frac{\not{x}}{\left(-x^2\right)^{d/2}} + \frac{m}{4\pi^{d/2}} \frac{\Gamma\left(d/2-1\right)}{\left(-x^2\right)^{d/2-1}} \tag{5}$$

Problem 3

After the Fourier transformation to momentum space, we expect that the correlation function $T_{\mu\nu}$ is transverse, that is

$$T_{\mu\nu} = \left(q_\mu q_\nu - q^2 g_{\mu\nu}\right) T(q^2). \tag{6}$$

Is this indeed the case? If not, what could be missing in this calculation?