Universität Regensburg, Institut für Theoretische Physik WS23
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Exercises on Quantum Chromodynamics problem sheet 7

Worksheet : Anomalous dimension.

Problem 1
Calculate the renormalization factors $Z_{\mathcal{O}}$, $[\mathcal{O}]_r = Z_{\mathcal{O}}\mathcal{O}$ for the operators

$$\mathcal{O} = \overline{q}(0)q(0), \qquad \mathcal{O}_\nu = \overline{u}(0)\gamma_\nu u(0).$$

To this end find the divergent part of the diagram

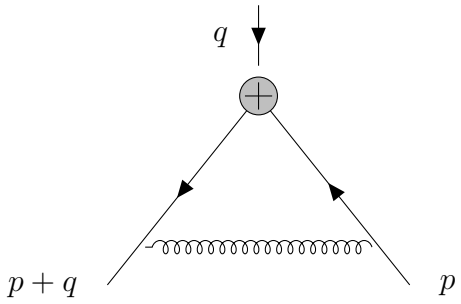

Figure 1: operator diagram

where the crossed circle stands for the operator. Convince yourself that the divergent part does not depend on the external momenta p, q and quark masses m_q. It means that in order to simplify the calculation one of the momenta and the quark masses can be put to zero. Why cannot we put all quantities to zero simultaneously, $q = p = m_q = 0$?

Problem 2
Calculate the anomalous dimension of the operators $\mathcal{O}$ and $\mathcal{O}_\nu$

$$\gamma_{\mathcal{O}} = \mu\frac{d}{d\mu}\log\left(Z_{\mathcal{O}}Z_q^-2\right)$$

where Z_q is the quark field renormalization constant (in Feynman gauge)

$$Z_q = 1 - \frac{\alpha_s}{\pi}\frac{1}{8\varepsilon}C_F + O(\alpha_s^2).$$

Check that the anomalous dimension of the operator $\mathcal{O}_\nu$ (the vector current) vanishes.