Universität Regensburg, Institut für Theoretische Physik WS23
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Exercises on Quantum Chromodynamics problem sheet 6

Worksheet : QCD vertex corrections.

On this exercise sheet you compute the corrections to the quark-gluon vertex in QCD.

Problem 1
Calculate the Quark contribution and extract the pole part at $D \to 4$.

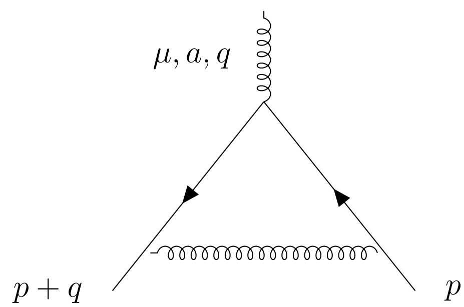

Figure 1: quark contribution

Problem 2
Calculate the Gluon contribution and extract the pole part at $D \to 4$.

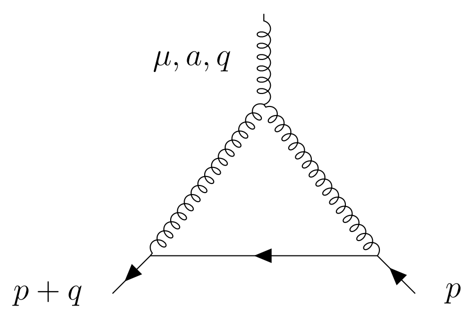

Figure 2: gluon contribution

Problem 3

QCD β-function: Earning your reward!

From our previous computation on sheet 5 we can extract

$$Z_2 = 1 - \frac{\alpha_s(\mu)}{4\pi}\frac{1}{\epsilon}C_F + \mathcal{O}(\alpha_s^2) \tag{1}$$

in the so called $\overline{\text{MS}}$-scheme.

From adaption from QED to QCD your computation for the electron self-energy yields

$$Z_3 = 1 + \frac{\alpha_s(\mu)}{4\pi}\frac{1}{\epsilon}(\frac{5}{3}C_A - \frac{4}{3}n_f T_f) + \mathcal{O}(\alpha_s^2) \tag{2}$$

Lastly, in this sheets computation you should have found

$$Z_1 = 1 - \frac{\alpha_s(\mu)}{4\pi}\frac{1}{\epsilon}(C_F + C_A) + \mathcal{O}(\alpha_s^2) \tag{3}$$

Take these results for granted, and recall that

$$g_0 = \mu^{\varepsilon} Z_1 Z_2^{-1} Z_3^{-\frac{1}{2}} g(\mu) \tag{4}$$

is scale-independent(!), to compute the first coefficient of the QCD β-function (β_0):

$$\beta(\alpha_s(\mu)) = \mu\frac{d\alpha_s(\mu)}{d\mu} = -\beta_0\frac{\alpha_s^2}{2\pi} + \mathcal{O}(\alpha_s^2) \tag{5}$$

in 4 dimentions ($\varepsilon \to 0$). You should obtain $\beta_0 = \frac{11}{3}C_A - \frac{4}{3}n_f T_f$.