Universität Regensburg, Institut für Theoretische Physik WS23
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Exercises on Quantum Chromodynamics
problem sheet 5

Worksheet : Gluon self-energy.

On this exercise sheet you will have to calculate the one-loop diagrams contributing to the gluon propagator in QCD. Good luck.

Problem 1
Calculate the Quark loop contribution, i.e. the diagram

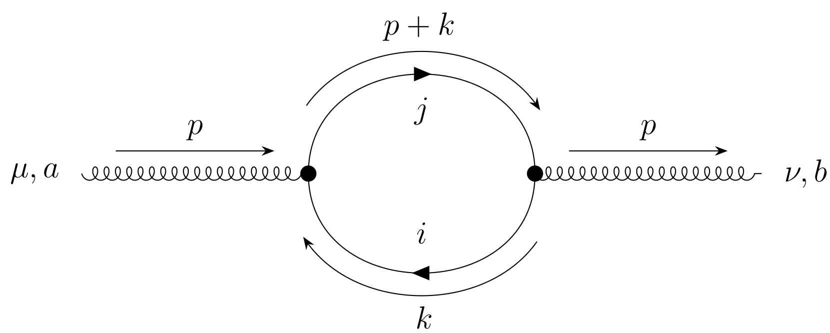

Figure 1: quark loop diagram

Or you take the result from QED lectures.
The QCD result can be obtained by minor modifications and is

$$\Pi^{a,b}_{\mu\nu} = -\frac{a_s}{\pi}\delta^{ab}\left(p^2 g^{\mu\nu} - p^\mu p^\nu\right)(4\pi)^{2-D/2}\frac{\Gamma(2-D/2)}{\left(-p^2\right)^{2-D/2}}\frac{\Gamma(D/2)}{\Gamma(D)}$$

Problem 2
Calculate the contribution of the gluon loop in dimension D and extract the pole part at $D \to 4$:

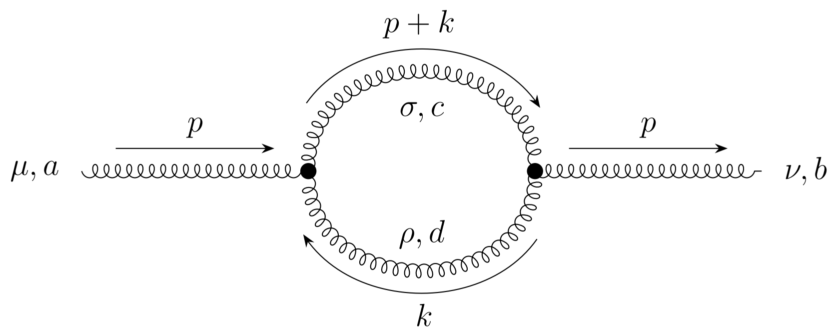

Figure 2: gluon loop diagram

Problem 3
Calculate the contribution of the ghost loop in dimension D and extract the pole part at $D \to 4$:

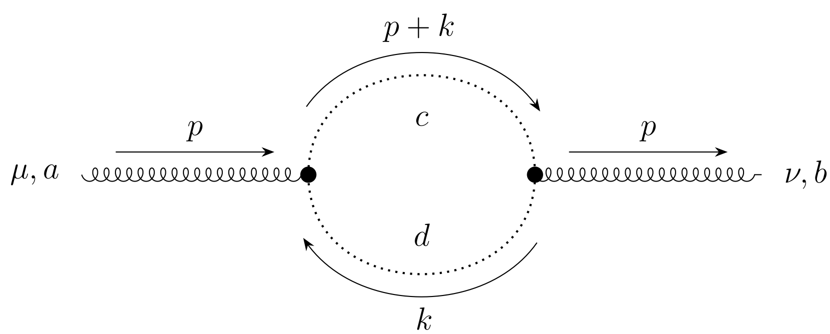

Figure 3: ghost loop diagram

Problem 4
Calculate the contribution of the diagram with a four-gluon coupling in dimension D.

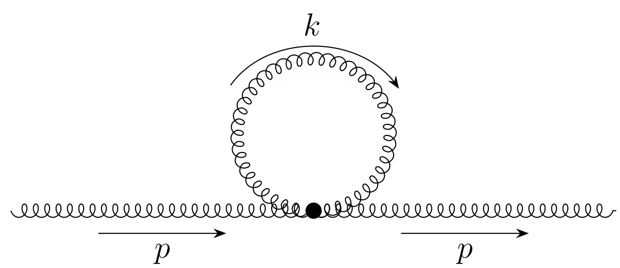

Figure 4: four-gluon-vertex loop diagram

This calculation involves the following sublety. The integrals over the type

$$\int \frac{d^D p}{p^2}$$

can be consistently put to zero in dimensional regularization despite the fact theat the integral diverges quadratically as $D \to 4$.
Let us consider this integral at $D \to 2$:

$$\int \frac{d^2p}{p^2}. \tag{1}$$

The integral diverges both for large and small p. The divergence for small p has nothing to do with ultraviolet renormalization and has to be treated separately. For instance, let us define the integral (1) as

$$\int \frac{d^2p}{p^2+m^2} \tag{2}$$

at $m^2 \to 0$. This integral diverges in the ultraviolet region only. We regularize it shifting $D = 2 \to 2 - 2\varepsilon$. Then

$$\int \frac{d^{2p-2\varepsilon}}{p^2+m^2} \approx \frac{1}{\varepsilon} m^{2\varepsilon} + \log m^2 \tag{3}$$

The pole in ε is removed by renormalization while the singularity at $m \to 0$ remains. This, so-called infrared singularity should be treated separately and its fate depends on the particular problem. Such singularities will be discussed later in the lectures.

Problem 4
Check that the sum of all diagrams is transverse, i.e. :

$$p^\mu \Pi^{\mu\nu} = 0.$$

Problem 5 An inspection shows that each of the diagrams considered above diverge not only at $D \to 4$ but also at $D \to 2$. It is easy to convince oneself that this divergence corresponds to a *quadratic* divergence of the diagrams, if they were calculated with an ultraviolet cutoff Λ_{UV}, that is each diagram actually contains terms $\approx \Lambda_{UV}^2$. Check that such divergences cancel in the sum of all diagrams; the diagram with the four-gluon vertex is important to ensure this cancellation despite the fact that it does not contribute to logrithmic renormalization. What would it mean if such quadratic divergences do not cancel?