Universität Regensburg, Institut für Theoretische Physik WS23
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# Exercises on Quantum Chromodynamics problem sheet 4

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*Worksheet : Renormalization.*

The aim of this exercise is to calculate a renormalized electron propagator. Consider the one-loop contribution to the electron self-energy:

$$\Sigma(p) = -e^2 \int \frac{d^4k}{(2\pi)^4 i} \gamma^\mu \frac{\not{p} - \not{k} + m_0}{m_0^2 - (p-k)^2 - i\epsilon} \gamma^\nu \frac{g_{\mu\nu}}{k^2 + i\epsilon} \tag{1}$$

**Problem 1**

Calculate the integral (1) using the cut-off regularization.
Show that the result can be written in the form

$$\Sigma(p) = \frac{e^2}{8\pi^2} \int_0^1 d\alpha [-(1-\alpha)\not{p} + 2m_0] \left( \log \frac{\alpha[m_0^2 - p^2(1-\alpha)]}{M^2} + 1 \right). \tag{2}$$

Notice that the terms depending on the cut-off parameter $M$ are linear in $m_0$ and $\not{p}$.

**Problem 2**

Represent the electron propagator in the form

$$S(p) = \frac{Z_2}{m - \not{p} + \Sigma^{(r)}(p)} \tag{3}$$

and find the wave function renormalization constant $Z_2$ and the renormalized self-energy $\Sigma^{(r)}$. You will find that even after imposing this divergence comes from small momentum $k \ll m, p$ region in the integral (1) and therefore has nothing to do with the small distances and will NOT be eliminated by renormalization procedure. In order to handle this divergence, the standard trick is tro introduce a small fictious photon mass:

$$\frac{1}{k^2 + i\varepsilon} \to \frac{1}{k^2 - \lambda^2 + i\varepsilon}$$

Here it is assumed that $\lambda^2 \ll m^2, p^2$ and should only be kept if the integration diverges otherwise. Use this trick to calculate the renormalization constant $Z_2(M, \lambda)$. Dependence on $\lambda$ seems embarassing. Try to guess what is the physical reason behind this problem (qualitatively) and how one might try to solve (avoid) it.

**Problem 3**

Find the physical mass $m$ in terms of the bare mass $m_0$.