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Exercises on Quantum Chromodynamics problem sheet 3

Worksheet 3: Gauge invariant Lagrangians and Feynman rules.

Problem 1

We consider the Lagrangian of a complex scalar field with a potential:

$$\mathcal{L} = \phi^*(x)\partial^2\phi(x) + g\left(\phi^*(x)\phi(x)\right)^2. \tag{1}$$

Now consider gauge transformations of the kind

$$\phi(x) \mapsto \phi'(x) = e^{i\alpha(x)}\phi(x),$$
$$\phi^*(x) \mapsto \left(\phi^*\right)'(x) = e^{-i\alpha(x)}\phi^*(x).$$

Using what you know from the lectures, rewrite the Lagrangian such that you obtain a new Lagrangian that is locally invariant with respect to the above gauge transformation.

Problem 2

Compute the Feynman rules for interactions between scalar fields and gauge fields of this gauge invariant Lagrangian!

You need to compute the corresponding correlation functions and define the Feynman rule for the vertex by truncating the external propagators.

This theory of scalar electrodynamics describes e.g. the interaction between photons and pions.

Problem 3

Now consider QCD. Your Lagrangian is

$$\mathcal{L}_{QCD} = \overline{\psi}(x)\left(i\not{D} - m\right)\psi(x) - \frac{1}{4}\left(F_a^{\mu\nu}(x)\right)^2, \tag{2}$$

with ferminon fields $\psi, \overline{\psi}$ and the self-interacting boson fields A^μ with

$$F_a^{\mu\nu}(x) = \partial^\mu A_a^\nu(x) - \partial_\nu A_a^\mu(x) + g f^{abc} A_b^\mu(x) A_c^\nu(x)$$

compute the Feynman rules for three interacting gluons.